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SENSITIVITY OF DIGITAL DUOBINARY OPTICAL RECEIVER

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RESUME

Cette article présente an analyse de la sensitivité d'un recepteur numérique optical dont la form d'onde à la sortie est "Duobinaire".

La dépendance de taux d'erreur moyenne sur les paramèters du systéme est présentée

SUMMARY

An analysis of the sensitivity of an optical receiver in a digital communication system with duobinary output pulse shaping is carried out. The dependence of the average error probability on system parameters is derived.

I. INTRODUCTION

The analysis and design of optical receivers with straight binary (NRZ and RZ) coding with raised-cosine output pulse shaping have been reported by many authors (1) - (3). In (4), an optical receiver in an optical transmission system using Manchester coding and raised cosine output pulse shaping is analyzed. A fiber optic system using a PIN FET high impedance receiver with duobinary output pulse shaping assuming full width rectangular received binary pulses is investigated in (5). In (6), a modified duobinary transmission system is investigated. The calculation of a duobinary system with avalanche photo diode (APD) is presented in (7) where the signal-dependent shot noise of the photo diode is considered in the manner explained in (8). The worst case signal energy in (7) required for a certain bit error rate is obtained assuming that the decision thresholds are halving the vertical eye diameter.

In this contribution, a digital binary optical communication system applying precoding and intensity modulation in the transmitter and duobinary output pulse shaping in the receiver is considered. The signal-dependent shot noise of the APD is calculated according to the method explained in (1). Decision thresholds are chosen so that they divide the range between voltage levels of the equalizer output according to their standard deviations. A formula is derived for the receiver sensitivity in terms of the average error probability, bit rate, extinction ratio, photodetector responsivity and average internal gain, input pulse shape, and other parameters of the receiver bias and input amplifier circuits.

II. SYSTEM CONSTRUCTION

Fig. 1 shows the block diagram of an optical fiber communication system applying duobinary equalizer in the receiver. The input binary sequence $\{a_j\}$ to be transmitted is first precoded into the binary sequence $\{b_j\}$ as follows

$$b_j = \begin{cases} 0 \text{ or } 1 \; ; & j = 0 \\ a_j \oplus b_{j-1} \; ; & j = 1,2,3,.. \end{cases} \qquad (1)$$

where $\oplus$ denotes modulo-2 addition. This precoding is only a transformation of the sequence $\{a_j\}$ into the sequence $\{b_j\}$ in order to avoid error propagation.

The light source is intensity-modulated by the binary sequence $\{b_j\}$ and the modulated beam is coupled to the optical fiber cable. At the end of the cable, the received modulated beam is coupled to the photo detector which detects the beam into an electric signal. After amplification and filteration by the duobinary equalizer the electric signal is sampled once per each pulse of the input sequence. Each sample is delivered to a threshold detector with two decision levels to discriminate between the three possible levels of the equalizer output. The decoder is then used to convert the 3-level sample into the binary estimate of the input sequence $\{\hat{a}_j\}$.

Fig. 2 shows the equivalent circuit of the receiver input blocks. For intensity modulation the binary digital pulse train incident on the photo detector can be described by

$$P(t) = \frac{1}{T_b} \sum_{-\infty}^{\infty} E_r \, s_{in}(t - rT_b) \qquad (2)$$

where $P(t)$ is the received optical power, and T_b is the bit duration. The parameter E_r, representing the r-th message digit, can

Fig. 1 Block diagram of an optical fiber communication system applying duobinary equalizer in the receiver.

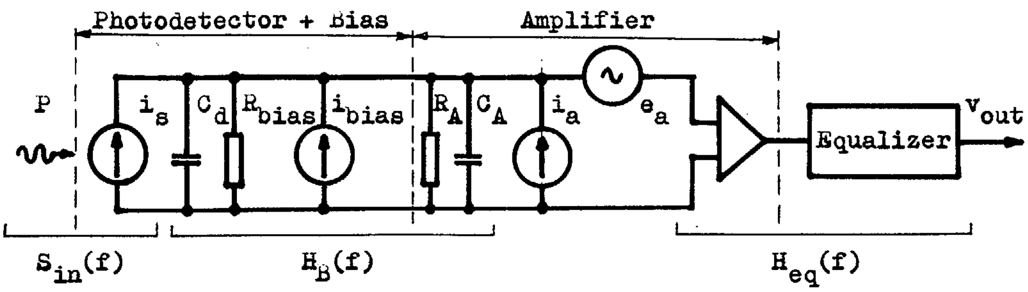

Fig. 2 Equivalent circuit of the receiver input blocks.

take the value E_{on} or E_{off} corresponding to a binary 1 or 0, respectively. If we assume that the non-negative received pulse shape $s_{in}(t)$ satisfies the integral

$$\frac{1}{T_b}\int_{-\infty}^{\infty} s_{in}(t)\,dt = 1 \tag{3}$$

then E_r represents the energy of the r-th pulse.

In response to the optical power $P(t)$, the photo detector emits primary electrons at an average rate $\lambda(t)$ per second given by

$$\lambda(t) = \frac{\eta}{h\nu}P(t) + \lambda_o \tag{4}$$

The average output current from the photo diode at time t resulting from the pulse train given in eq. (2) is

$$\begin{aligned} I_s(t) &= \lambda(t)\,q\,\bar{M} \\ &= R_o\,\bar{M}\,P(t) + q\,\bar{M}\,\lambda_o \end{aligned} \tag{5}$$

where $\bar{M}$ is the average detector internal gain, q is the electron charge, and R_o is the photo detector responsivity ($= \eta q/h\nu$).

This current is then amplified and filtered to produce a mean voltage at the equalizer output (neglecting dc components) given by

$$\bar{v}_{out}(t) = R_o\,\bar{M}\,P(t)*h_B(t)*h_{eq}(t) \tag{6}$$

where "*" denotes convolution, $h_B(t)$ is the impulse response of the bias and amplifier input circuit, and $h_{eq}(t)$ is the impulse response of the equalizer. The equalizer output voltage is of the form

$$v_{out}(t) = \sum_{-\infty}^{\infty} V_r\,s_{out}(t-rT_b) + v_n(t) \tag{7}$$

where $v_n(t)$ represents the noises. In the duobinary case, the output pulse shape $s_{out}(t)$ and its Fourier transform are

$$s_{out}(t) = \frac{4}{\pi}\,\frac{\cos(\pi t/T_b)}{1-4(t/T_b)^2} \tag{8}$$

$$S_{out}(f) = \begin{cases} 2T_b\cos(\pi f T_b), & |f|<1/2T_b \\ 0, & |f|\geqslant 1/2T_b \end{cases} \tag{9}$$

III. DETECTION, DECODING, AND ERROR

For detecting the r-th symbol we sample at the instant $t_s = (r-\frac{1}{2})T_b$

$$v_{out}(t_s) = V_{r-1}+V_r+v_n = V_{signal}+v_n \tag{10}$$

Each of the quantities V_{r-1} and V_r can be either $V_{off} = \int E_{off}$ or $V_{on} = \int E_{on}$. Thus, V_{signal} can be $2V_{off}$, $V_{off}+V_{on}$, or $2V_{on}$. If the symbols "0" and "1" have equal probabilities, then the probabilities of $2V_{off}$, $V_{off}+V_{on}$, and $2V_{on}$ are ¼,½,¼, respectively. Two threshold levels are applied; V_{th1} and V_{th2} where $2V_{off}<V_{th1}<V_{off}+V_{on}<V_{th2}<2V_{on}$. The combined decision and decoding rule is to select "1" if $V_{th1}<v_{out}<V_{th2}$ and select "0" otherwise. The average error probability is

$$P_e = \tfrac{1}{4}P_e/2V_{off}+\tfrac{1}{2}P_e/(V_{off}+V_{on})+\tfrac{1}{4}P_e/2V_{on} \tag{11}$$

Assume that the equalizer output is Gaussian with variances σ_1^2, σ_2^2, and σ_3^2 corresponding to $2V_{off}$, $V_{off}+V_{on}$, and $2V_{on}$ respectively. Let the threshold levels satisfy the following relations

$$(V_{th1}-2V_{off})/\sigma_1=(V_{off}+V_{on}-V_{th1})/\sigma_2=Q_1 \tag{12}$$

$$(2V_{on}-V_{th2})/\sigma_3=(V_{th2}-V_{off}-V_{on})/\sigma_2=Q_3 \quad (13)$$

Therefore, we have

$$Q_1 = (V_{on}-V_{off})/(\sigma_1+\sigma_2) \quad (14)$$

$$Q_3 = (V_{on}-V_{off})/(\sigma_3+\sigma_2) = c_\sigma Q_1 \quad (15)$$

$$c_\sigma = (\sigma_1+\sigma_2)/(\sigma_3+\sigma_2)=(\dot{\sigma}_1+\dot{\sigma}_2)/(\dot{\sigma}_3+\dot{\sigma}_2) \quad (16)$$

where

$$\sigma_1/\dot{\sigma}_1=\sigma_2/\dot{\sigma}_2=\sigma_3/\dot{\sigma}_3=\xi\sigma_E \quad (17)$$

$$\sigma_E = (q/R_o)/\overline{M} \quad (18)$$

Substituting, we get

$$P_e = 0.75\ \psi_{c_\sigma}(Q_1/\sqrt{2}) \quad (19)$$

where

$$\psi_{c_\sigma}(x) = \tfrac{1}{2}\left[\mathrm{erfc}(x) + \mathrm{erfc}(c_\sigma x)\right] \quad (20)$$

$$Q_1 = (P_{in}/\sigma_E R_b)(1-\varepsilon)/(\dot{\sigma}_1+\dot{\sigma}_2) \quad (21)$$

$\varepsilon(=E_{off}/E_{on})$ is the extinction ratio of the light source, R_b $(=1/T_b)$ is the bit rate of the input sequence, and P_{in} $(=E_{on}/T_b=E_{on}R_b)$ is the average optical power at the receiver input corresponding to a "1".

IV. CALCULATION OF NOISE COMPONENTS

The noise voltage $v_n(t)$ at the equalizer output can be represented by

$$v_n(t) = v_s(t)+v_{R_{bias}}(t)+v_I(t)+v_E(t) \quad (22)$$

which is the sum of the shot noise voltage due to $i_s(t)$, the thermal noise voltage due to i_{bias}, the output noise voltages due to $i_a(t)$ and $e_a(t)$, respectively. The amplifier noise sources are assumed independent of each other and Gaussian in statistics. At the equalizer output, the mean square noise voltage v_n^2 is given by

$$\overline{v_n^2} = \overline{v_s^2} + \overline{v_{R_{bias}}^2} + \overline{v_I^2} + \overline{v_E^2} \quad (23)$$

Different noise components are calculated as follows :

$$\overline{v_{R_{bias}}^2}=(2k\theta/R_{bias})\int_{-\infty}^{\infty}|H_B(f)H_{eq}(f)|^2 df \quad (24)$$

$$\overline{v_I^2} = S_I\int_{-\infty}^{\infty}|H_B(f)H_{eq}(f)|^2\, df \quad (25)$$

$$\overline{v_E^2} = S_E\int_{-\infty}^{\infty}|H_{eq}(f)|^2\, df \quad (26)$$

where k, θ, S_I, and S_E are Boltzmann's constant, absolute temperature, spectral density of $i_a(t)$, and spectral density of $e_a(t)$, respectively. In terms of the Fourier transform of input and output pulse shapes, the above equations can be rewritten as follows :

$$\overline{v_{R_{bias}}^2}=(2k\theta/R_{bias})(\xi^2\sigma_E^2/q^2R_b)I_2 \quad (27)$$

$$\overline{v_I^2} = S_I(\xi^2\sigma_E^2/q^2R_b)I_2 \quad (28)$$

$$\overline{v_E^2} = S_E(\xi^2\sigma_E^2/q^2R_b)\left[I_2/R_t^2+(2\pi C_tR_b)^2I_3\right] \quad (29)$$

where

$$I_2 = \int_{-\infty}^{\infty}\left|S'_{out}(x)/S'_{in}(x)\right|^2\, dx \quad (30)$$

$$I_3 =\int_{-\infty}^{\infty}\left|S'_{out}(x)/S'_{in}(x)\right|^2\, x^2\, dx \quad (31)$$

$$R_t = R_{bias}R_A/(R_{bias}+R_A) \quad (32)$$

$$C_t = C_d + C_A \quad (33)$$

$$S'_{in}(x) = R_b\, S_{in}(xR_b) \quad (34)$$

$$S'_{out}(x) = R_b\, S_{out}(xR_b) \quad (35)$$

The shot noise term is

$$\overline{v_s^2}(t) =\int_{-\infty}^{\infty} q^2\overline{M^2}\left[(R_o/q)\sum_r E_rR_bs_{in}(t'-rT_b) +\lambda_o\right] h_I^2(t-t')\, dt' \quad (36)$$

where

$$h_I(t) = F^{-1}\left[H_B(f)\ H_{eq}(f)\right] \quad (37)$$

When detecting the zero-th symbol "E_o", we sample the equalizer output voltage at the instant $t_o = -T_b/2$

$$v_{out}(t_o) = V_{-1} + V_o + v_n(t_o) \quad (38)$$

Equation (36) shows that the first term of the shot noise depends on the detected symbol E_o and its neighboring symbols. Since $s_{in}(t)$ is positive for all t, the worst case shot noise, for certain values of E_o and E_{-1}, occurs when all the E_r (except E_{-1} and E_o) assume the larger of the two possible values (E_{on} and E_{off}).

Converting the integral from time domain to frequency domain and summing up the noise components, we get

$$\overline{v_n^2}(t_o)=(\rho^2\sigma_E^2\overline{M^2}R_o/qR_b)\int_{-\infty}^{\infty}\sum E_r R_b \exp[-j\pi x(2r+1)]\cdot S_1'(x)\,dx+\rho^2\sigma_E^2 Z \quad (39)$$

where

$$S_1'(x) = S_{in}'(x)\left[\frac{S_{out}'(x)}{S_{in}'(x)} * \frac{S_{out}'(x)}{S_{in}'(x)}\right] \quad (40)$$

$$Z = \frac{1}{q^2R_b}\left(\frac{2\,k\,\Theta}{R_{bias}} + S_I + \frac{S_E}{R_t^2} + q^2\,\overline{M^2}\,\lambda_o\right) I_2 + (R_b/q^2)(2\pi C_t)^2 S_E I_3 \quad (41)$$

The maximum value of the integral in the first term of $\overline{v_n^2}(t_o)$, which corresponds to the worst case, is obtained by setting all E_r (except E_{-1} and E_o) equal to E_{on}. Substituting, we get

$$\overline{v_n^2}(t_o)_{max}=\rho^2\sigma_E^2\left\{\mathcal{H}\left[\Sigma_1+\left(\frac{E_o+E_{-1}}{E_{on}}-2\right)I_{1c}+\left(\frac{E_o-E_{-1}}{E_{on}}\right)I_{1s}\right]+Z\right\} \quad (42)$$

where

$$\mathcal{H} = \overline{M^2}(R_o/q)(P_{in}/R_b) \quad (43)$$

$$\Sigma_1 = \sum_{r=-\infty}^{\infty}(-1)^r\,S_1'(r) \quad (44)$$

$$I_{1c} = \int_{-\infty}^{\infty} S_{1R}'(x)\cos\pi x\,dx \quad (45)$$

$$I_{1s} = \int_{-\infty}^{\infty} S_{1I}'(x)\sin\pi x\,dx \quad (46)$$

$S_{1R}'(x)$ and $S_{1I}'(x)$ are the real and imaginary parts of $S_1'(x)$.

The variances σ_1^2, σ_2^2, and σ_3^2 are obtained by setting (E_{-1},E_o) in eq. (42) equal to (E_{off},E_{off}), (E_{off},E_{on}), and (E_{on},E_{on}), respectively. Thus, we have

$$\sigma_1'^2 = \mathcal{H}\left[\Sigma_1 - 2(1-\varepsilon)I_{1c}\right] + Z \quad (47)$$

$$\sigma_2'^2 = \mathcal{H}\left[\Sigma_1 - (1-\varepsilon)I_{1c} + (1-\varepsilon)\,|I_{1s}|\right] + Z \quad (48)$$

$$\sigma_3'^2 = \mathcal{H}\Sigma_1 + Z \quad (49)$$

The quantities Σ_1, I_{1c}, I_{1s}, I_2, and I_3 depend only on $S_{in}'(x)$ and $S_{out}'(x)$. In our case (duobinary), $S_{out}'(x)$ is given by

$$S_{out}'(x) = \begin{cases} 2\cos(\pi x), & |x| < \tfrac{1}{2} \\ 0, & |x| \geqslant 0 \end{cases} \quad (50)$$

Since $S_{out}'(x) = 0$ for $|x| \geqslant 0.5$, also will be $S_{out}'(x)/S_{in}'(x)$. Thus ,

$$[S_{out}'(x)/S_{in}'(x)] * [S_{out}'(x)/S_{in}'(x)] = 0, \quad |x| \geqslant 1.$$

Consequently, $S_1'(x) = 0$ for $|x| \geqslant 1$ and Σ_1 reduces to

$$\Sigma_1 = S_1'(0) = S_{in}'(0)\,I_2 = I_2 \quad (51)$$

V. RESULTS

The sensitivity of the considered digital duobinary optical receiver can be calculated using the formula

$$P_{in} = Q_1\,\sigma_E(\sigma_1' + \sigma_2')\,R_b/(1-\varepsilon) \quad (52)$$

where

$$Q_1 = \sqrt{2}\,\psi_{c_G}^{-1}(\tfrac{4}{3}\,P_e) \quad (53)$$

When the system employs a PIN photodiode in the receiver instead of the APD, we set

$\overline{M^2} = 0$ and $\overline{M} = 1$.

The above results reduce to

$$P_{in} = \left(\frac{q}{R_o}\right)\left(\frac{R_b}{1-\varepsilon}\right)\,2\sqrt{2Z}\,\mathrm{erfc}^{-1}(\tfrac{4}{3}\,P_e) \quad (54)$$

VI. CONCLUSION

We have presented an analysis of the receiver design for a digital optical fiber communication system with duobinary output pulse shaping. Assuming that the equalizer output voltage is Gaussian, expressions are obtained to predict the receiver sensitivity and the average error probability in terms of Fourier transform of the input pulse shape, bit rate, and other system parameters.

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