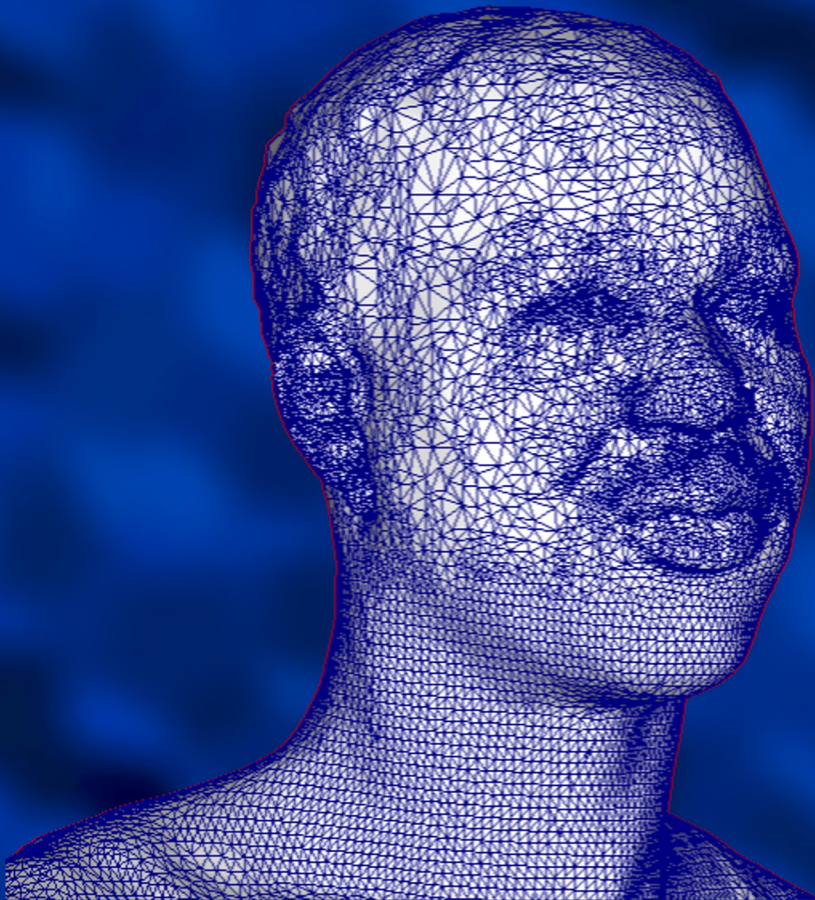


ECCV Tutorial

Mesh Processing

Parameterization



Bruno Lévy
INRIA - ALICE

Outline

- 1. Introduction - motivation
- 2. Simple objects
- 3. More complicated objects
- 4. Spectral methods

Epilogue

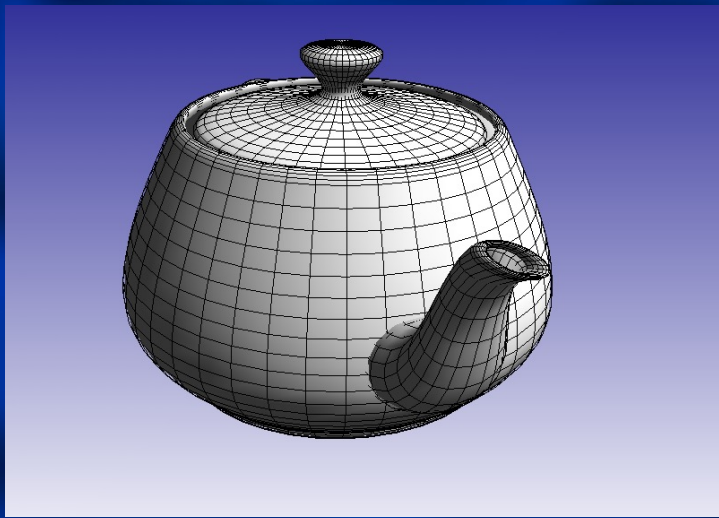
Introduction

Why parameterization ?

- Texture mapping
- ... and more

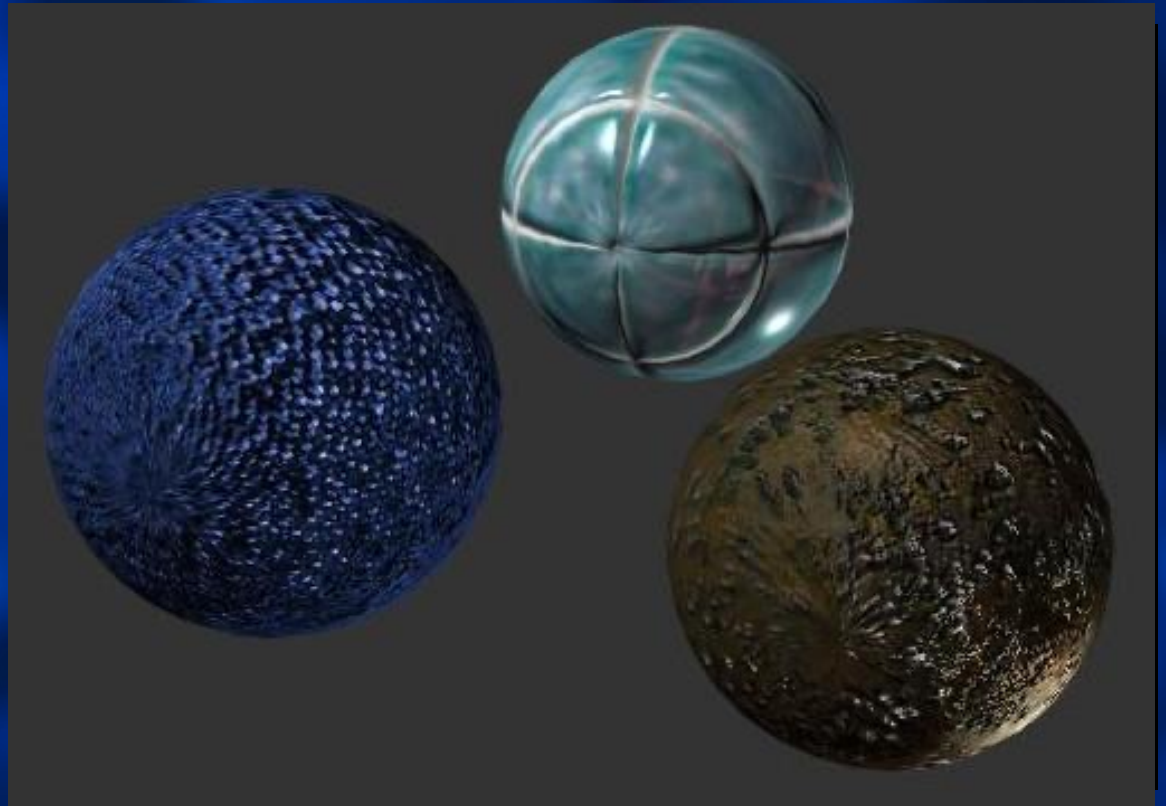
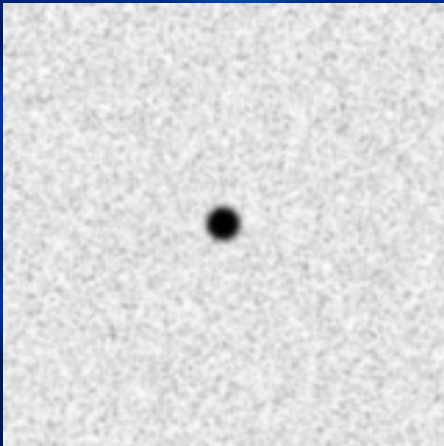
1. Introduction

Ed Catmull - 1974



1. Introduction

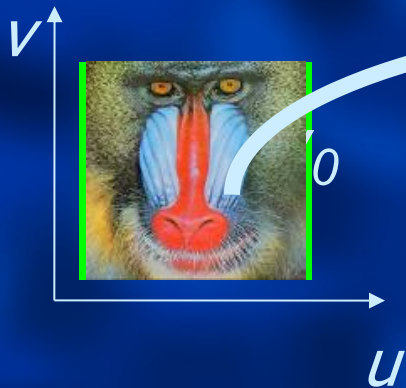
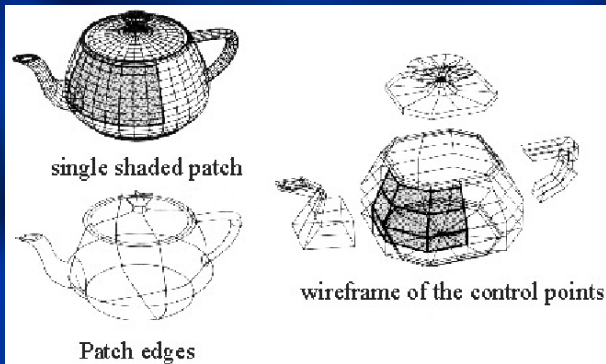
Jim Blinn - 1978



Bump mapping

1. Introduction

Splines: we got a parameterization



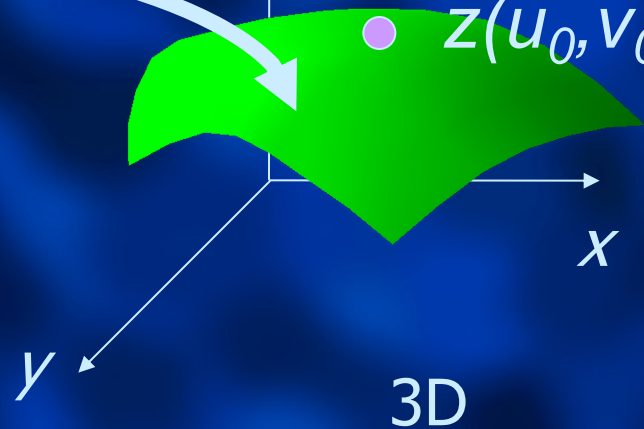
2D-Parameter
space



$$x(u_0, v_0)$$

$$y(u_0, v_0)$$

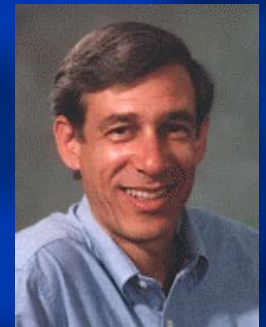
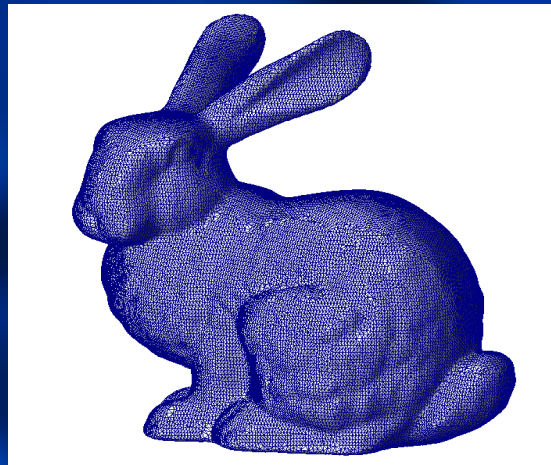
$$z(u_0, v_0)$$



1. From Newell's teapot to Stanford's Bunny



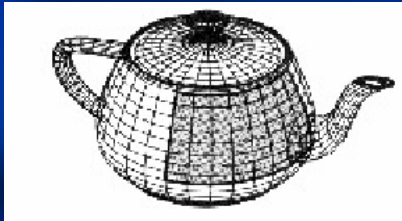
Martin Newell



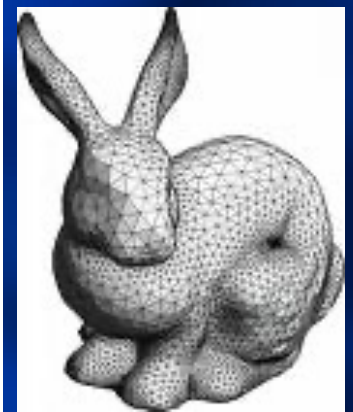
[Turk and Levoy 94]

Q: How can we do texture mapping on the bunny ?

1. From Newell's teapot to Stanford's Bunny



Parametric
form
Natural (u,v)



Mesh
No (u,v) !

Alice's adventures in wonderland

1. Introduction

Other motivations

Creating a CAD model from a real car ...



1970's: purely manual acquisition (Y. Sutherland)

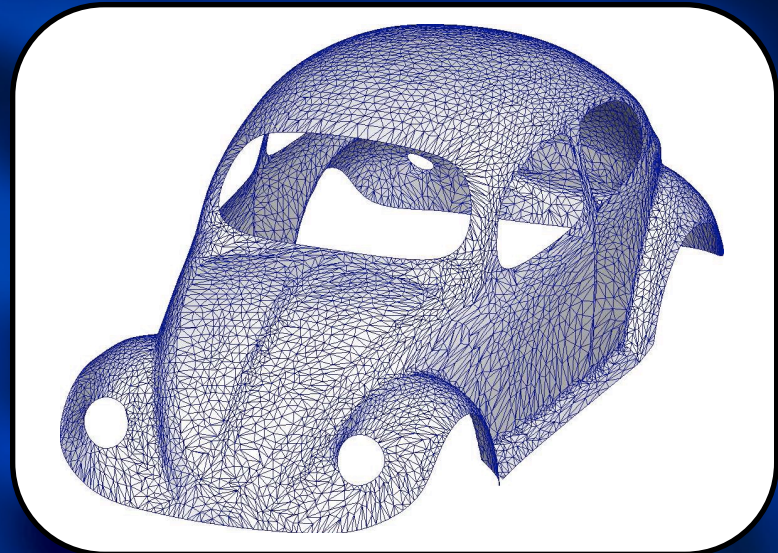
1. Introduction

Other motivations

Creating a CAD model from a real car ...



3D scanner



Reconstructed model

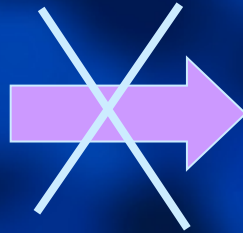
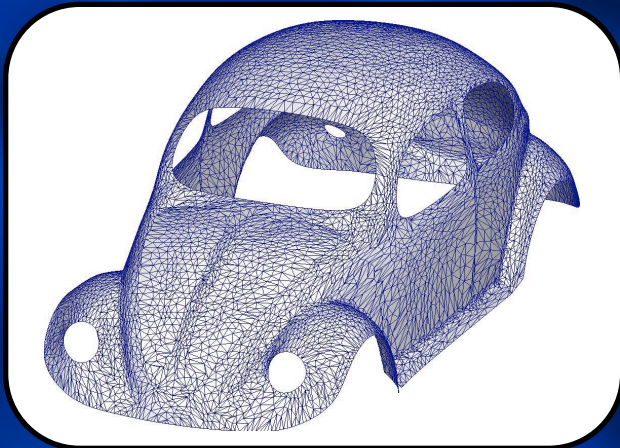
[Levoy & Turk], [Boissonnat], ...

1 million vertices, 2 millions triangles:
Cannot be used by CAD/CAM software

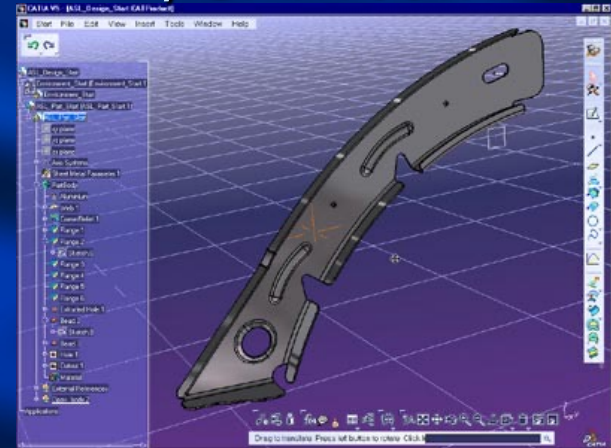
1. Introduction

Other motivations

output of the 3D scanner



CAD/CAM software



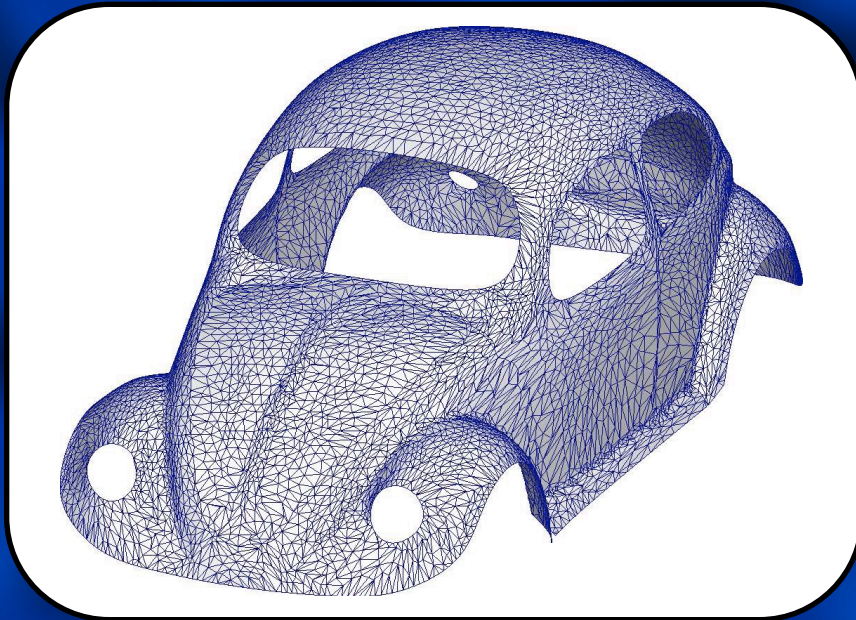
The **representation** is not the right one,
CAD/CAM needs ...
... **equations** instead of **sampling**

How can we find **the equation** of this car ?

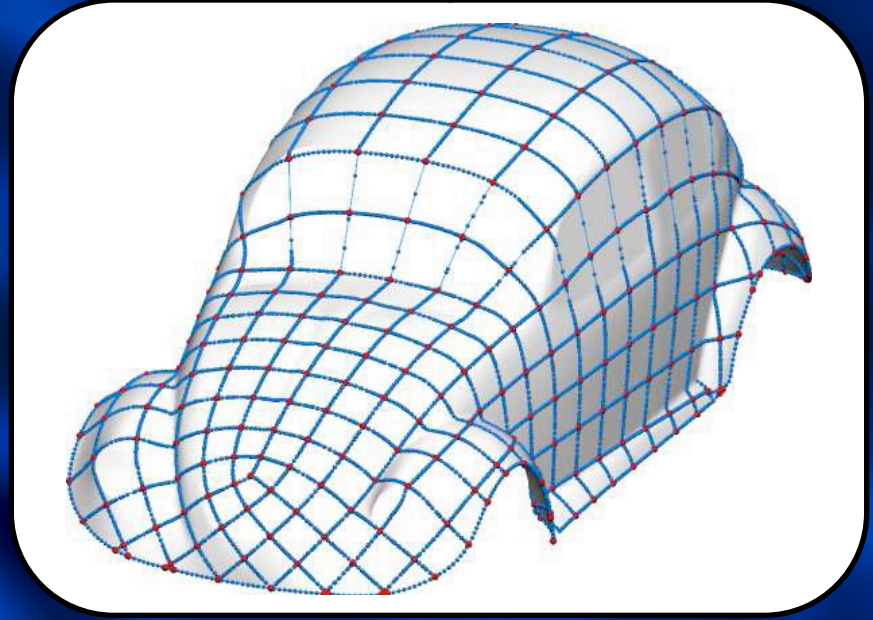
1. Introduction

More Motivations

output of the scanner



CAD/CAM representation



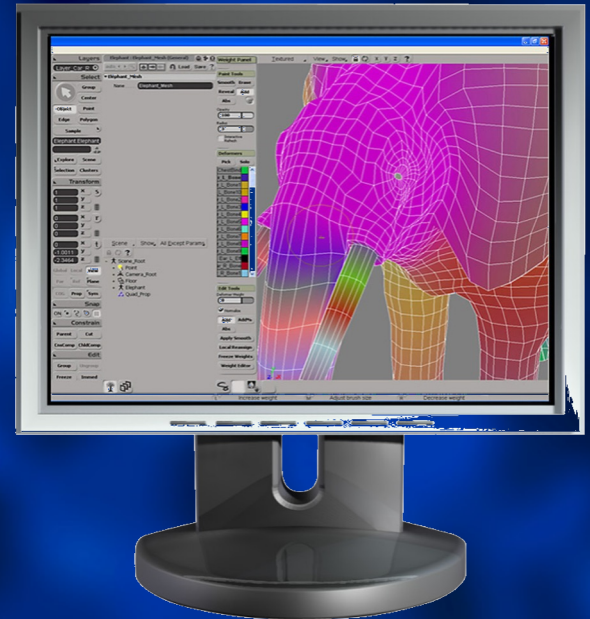
1 million points $\rightarrow$ 100 Splines (bicubic equations)
How can we find **the equation** of this car ?

1. Introduction

Square pieces in round holes ...



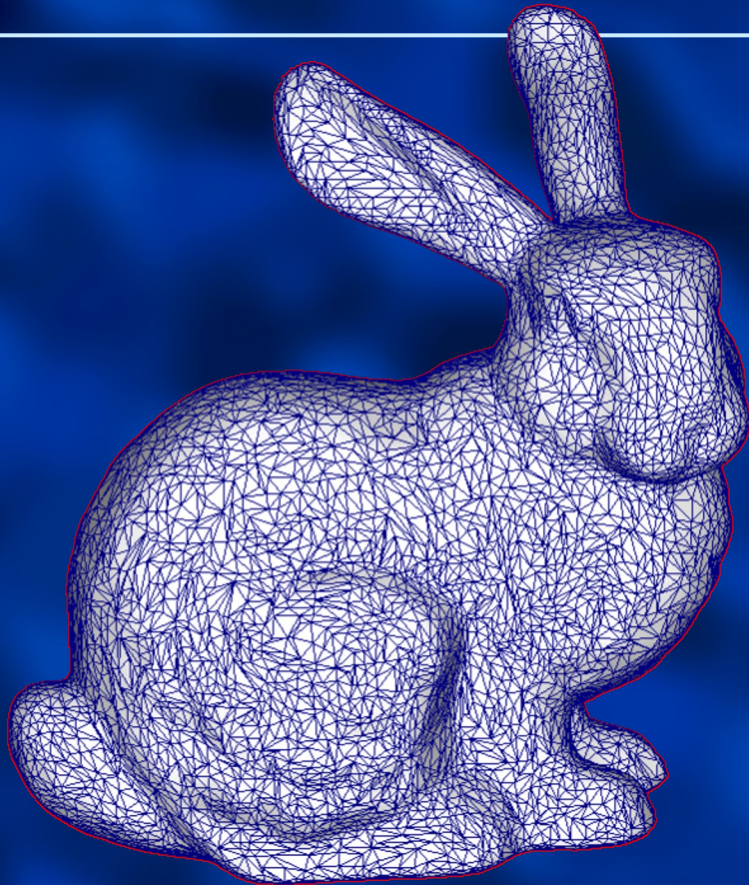
Scanner 3D
Acquisition



3D modeling
Simulation
Visualization ...

Not the right
representation !!!

2. Texture mapping

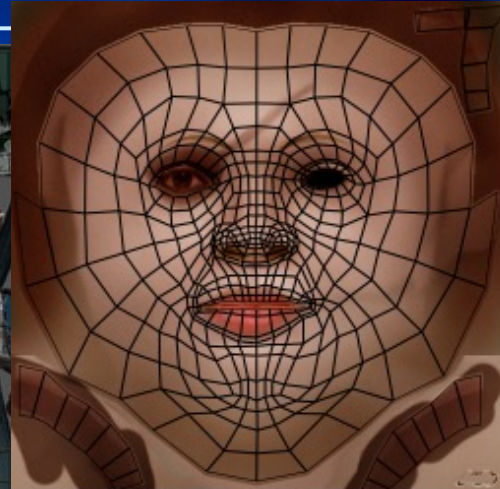


What is the equation
of this bunny ?



What is the point
asking ?

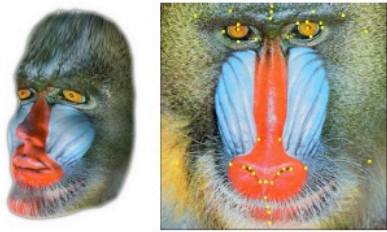
1. Can we play Catmull/Blinn game with meshes ?



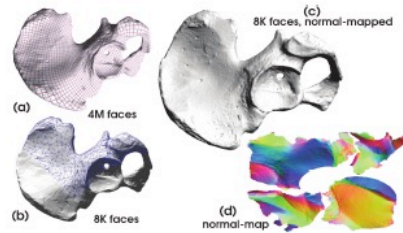
Texture
atlas
(Alias | Wavefront
Maya)



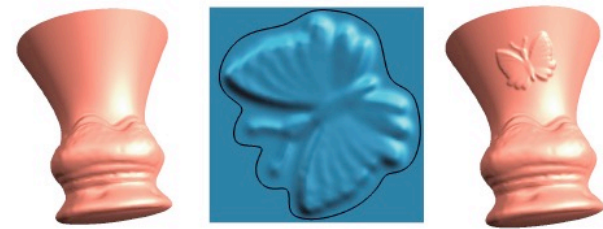
1. Other Applications



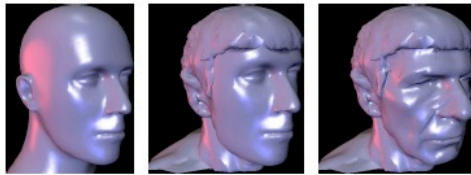
Texture Mapping



Normal Mapping



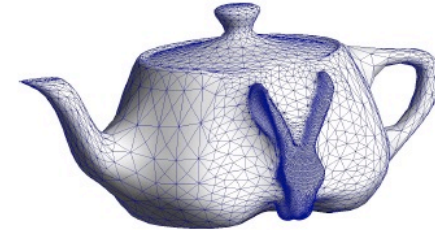
Detail Transfer



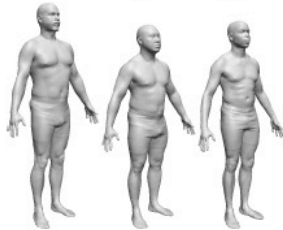
Morphing



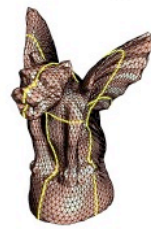
Mesh Completion



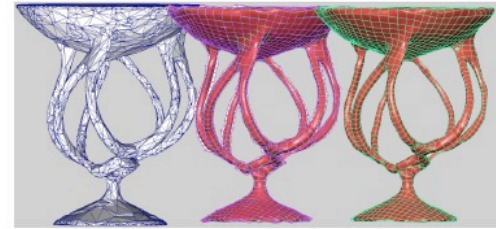
Editing



Databases



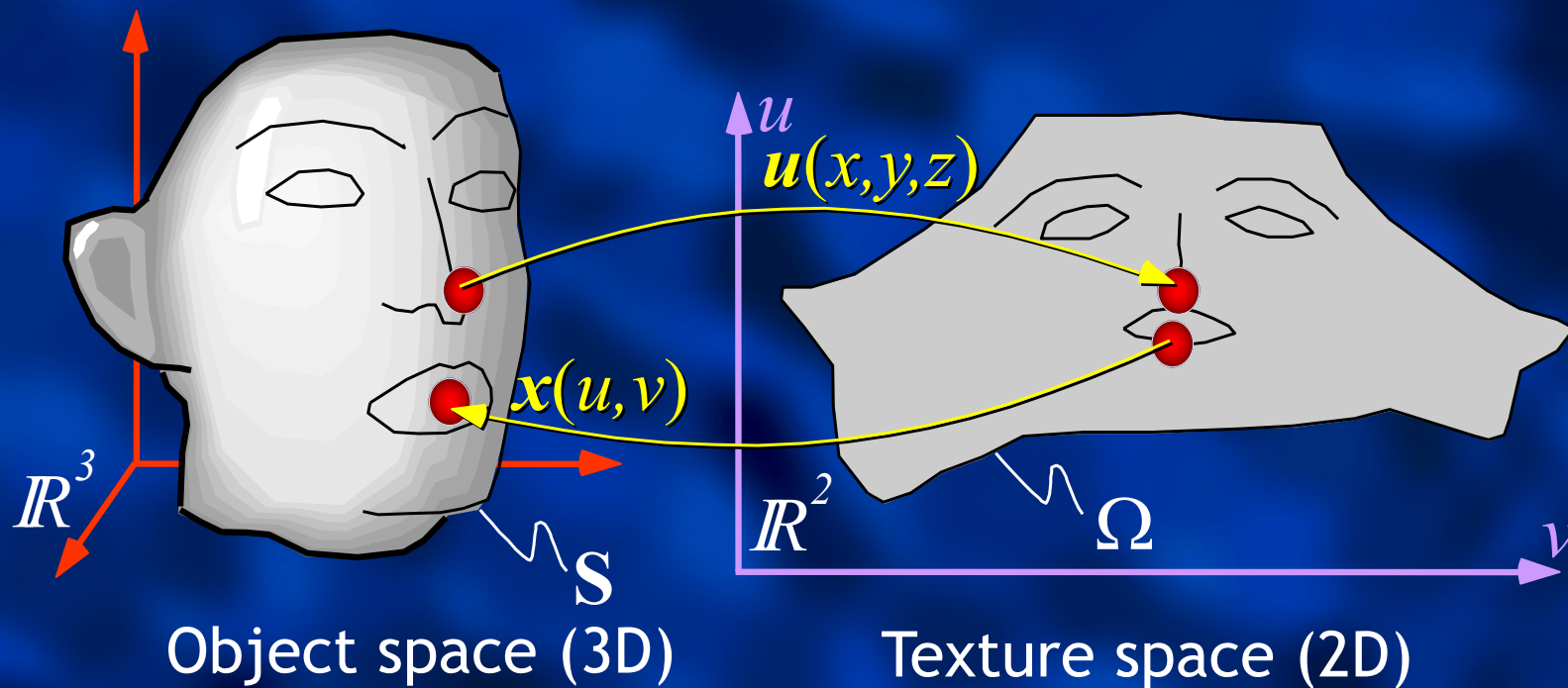
Remeshing



Surface Fitting

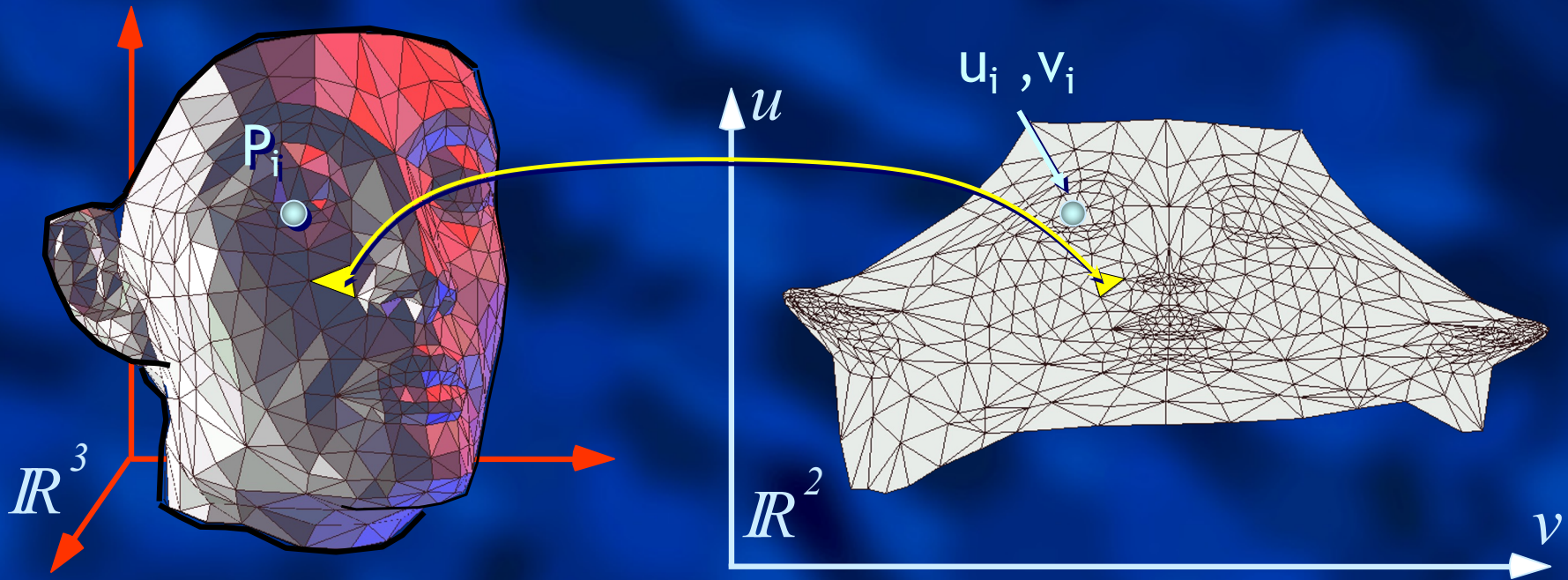
2. Simple Objects

Notion of parameterization $x(\dots)$



2. Simple Objects

Triangulated surfaces



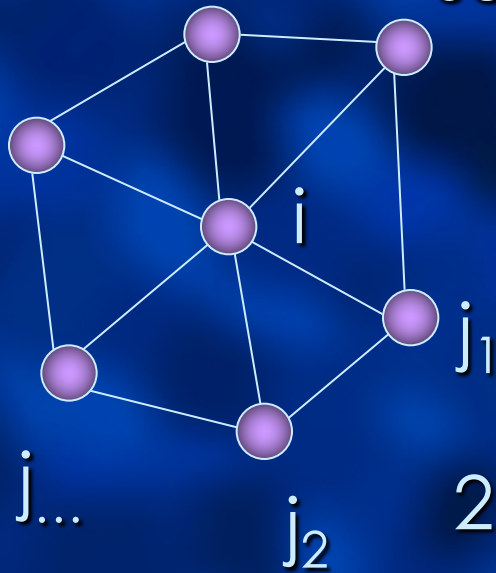
Object space (3D)

Texture space (2D)

Q: How can we find all these (u_i, v_i) 's ?

2. Simple Objects

Tutte theorem (1960's)



1) we got some coefficients $a_{i,j}$ such that

$$(u_i, v_i) = \sum_{j \in N_i} a_{i,j} (u_j, v_j)$$

$$\forall i, j \quad a_{i,j} > 0$$

$$a_{i,i} = - \sum a_{i,j}$$

2) The border is mapped to a convex polygon

Then we got a valid mapping

2. Simple objects

Floater's idea

- Tutte's theorem **characterizes** a family of valid mappings
- Why not using it to **construct** a valid mapping ?

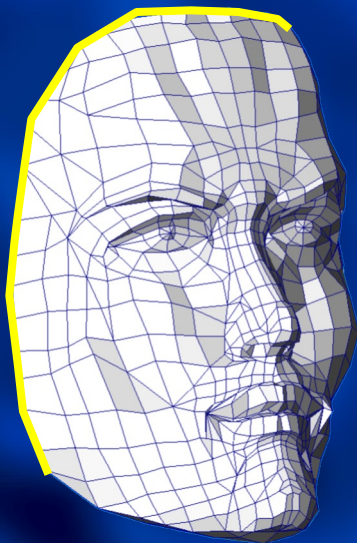


M. Floater

2. Simple Objects

Floater's idea

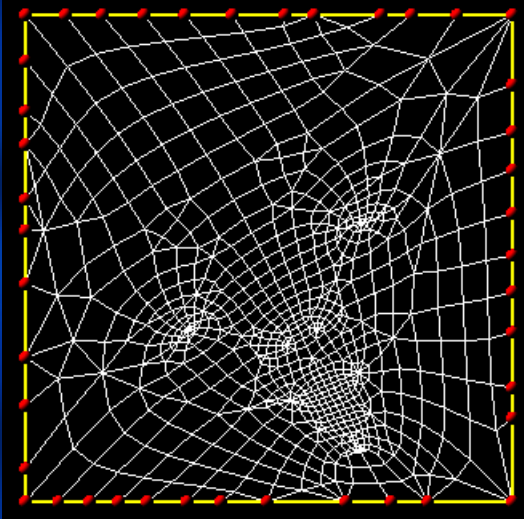
- 1. Map the border to a convex polygon
 - 2. Solve $A u = u'$
 - 3. Solve $A v = v'$
- Q: How do you choose the coeffs. of $A = [a_{i,j}]$



M. Floater

2. Simple Objects

Choosing the weights a_{ij}



$$a_{ij} = 1 \quad \forall i \neq j$$



Problem: distortion !!!

2. Simple Objects minimizing distortion

- [Greiner et.al]: Variational principles for geometric modeling with Splines
PDEs for geometric optimization



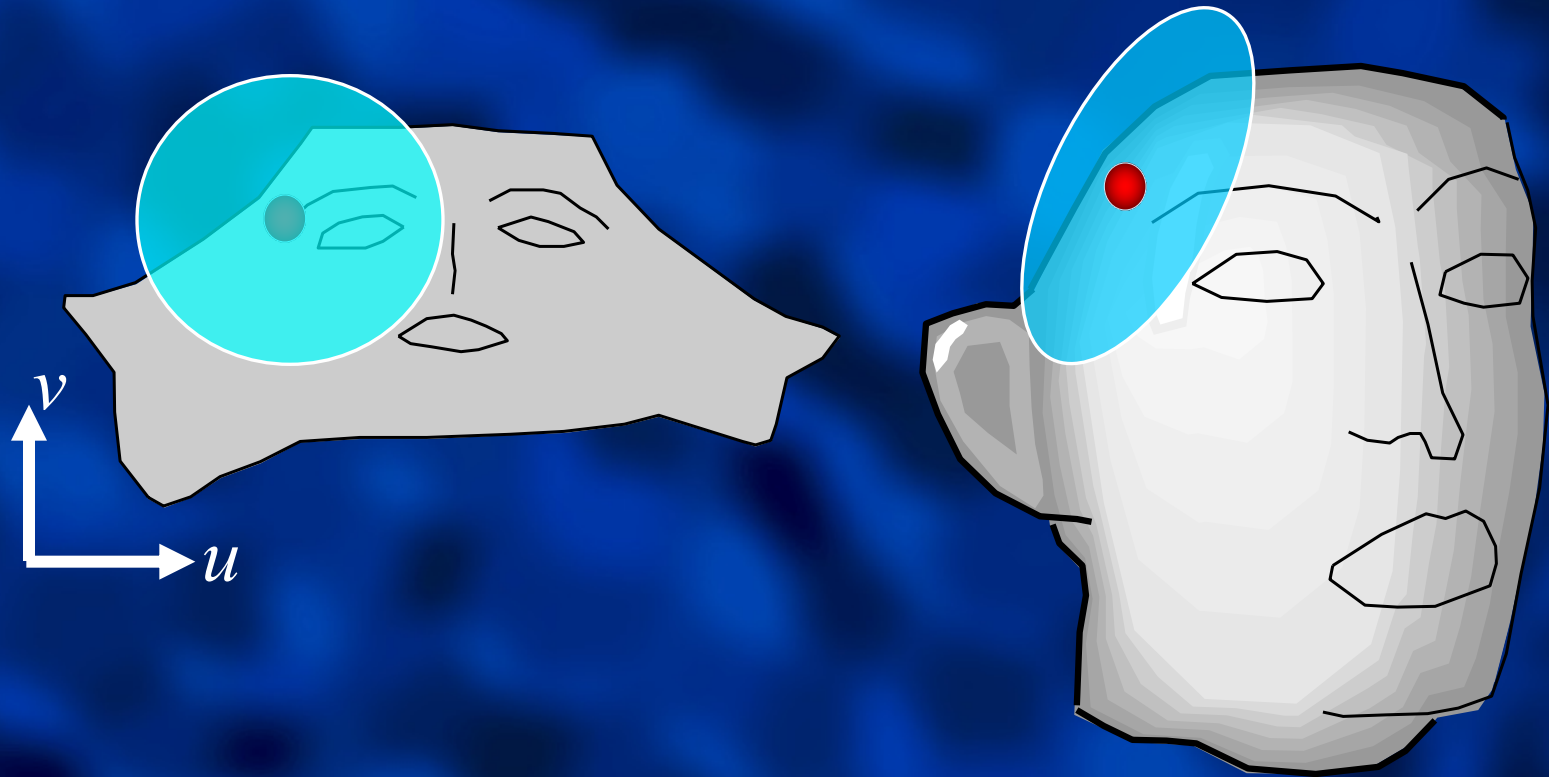
Kai Hormann



Can we port this principle to
the discrete setting ? [Hormann & Greiner]

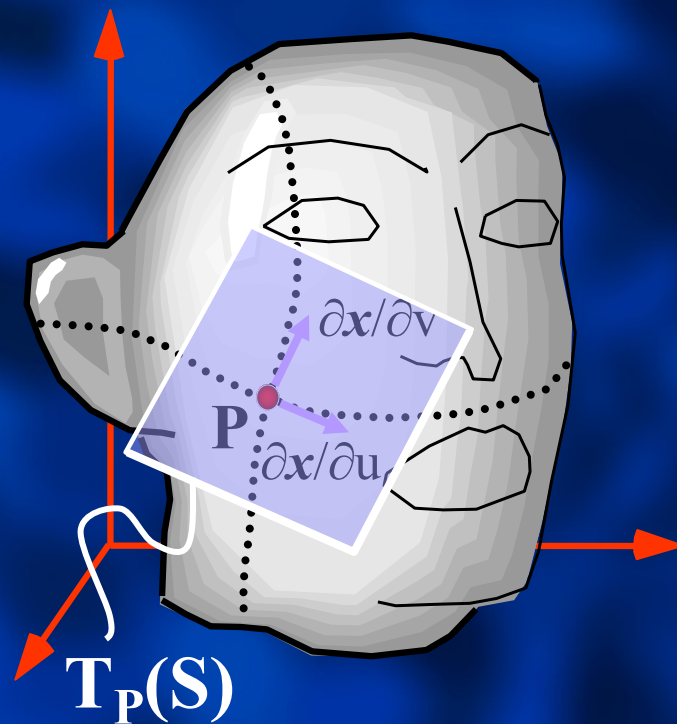
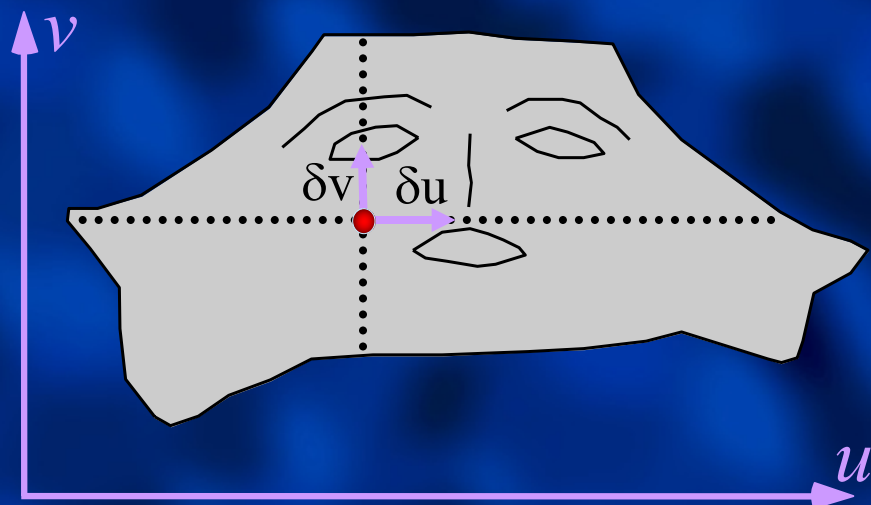
2. Simple Objects

Anisotropy



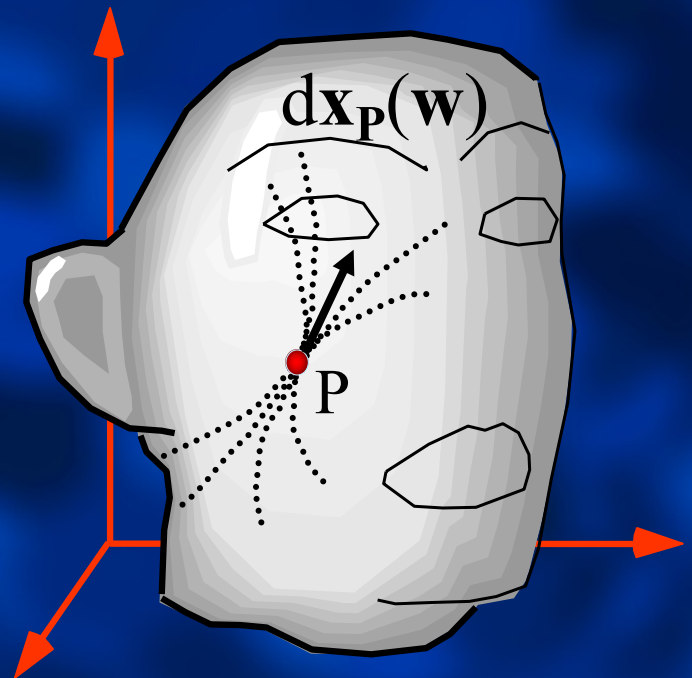
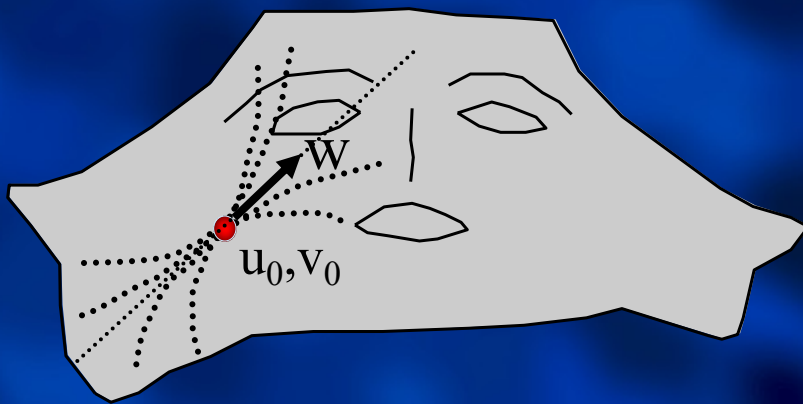
Geometry of $T_p(S)$

Partial derivatives of $x(\dots)$



Geometry of $T_p(S)$

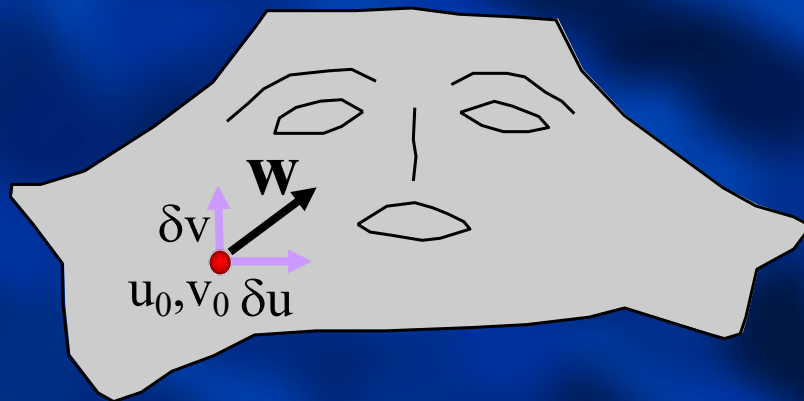
Differential dx_p ; directional derivatives



$$dx_p(w) = \partial / \partial t \left(x((u_0, v_0) + t.w) \right)$$

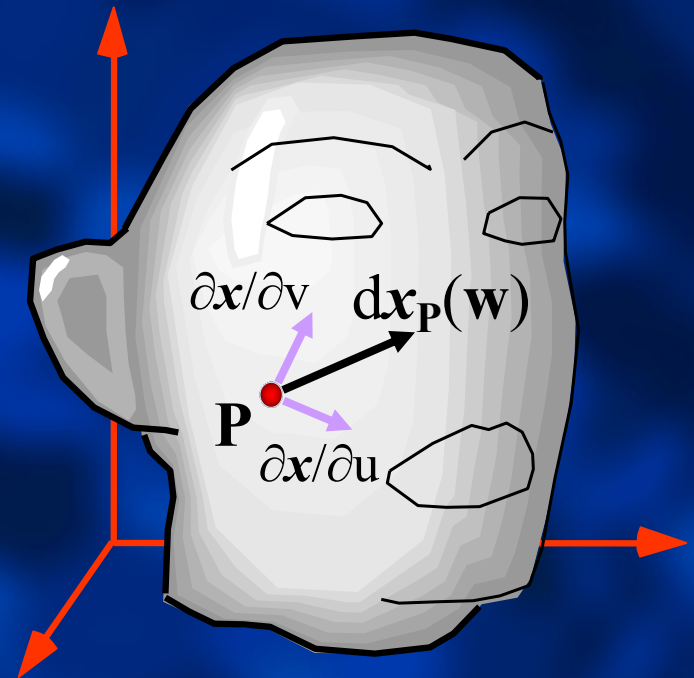
Geometry of $T_p(S)$

Jacobian Matrix J_P



$$J_P = \begin{bmatrix} \partial x / \partial u & \partial x / \partial v \\ \partial y / \partial u & \partial y / \partial v \\ \partial z / \partial u & \partial z / \partial v \end{bmatrix}$$

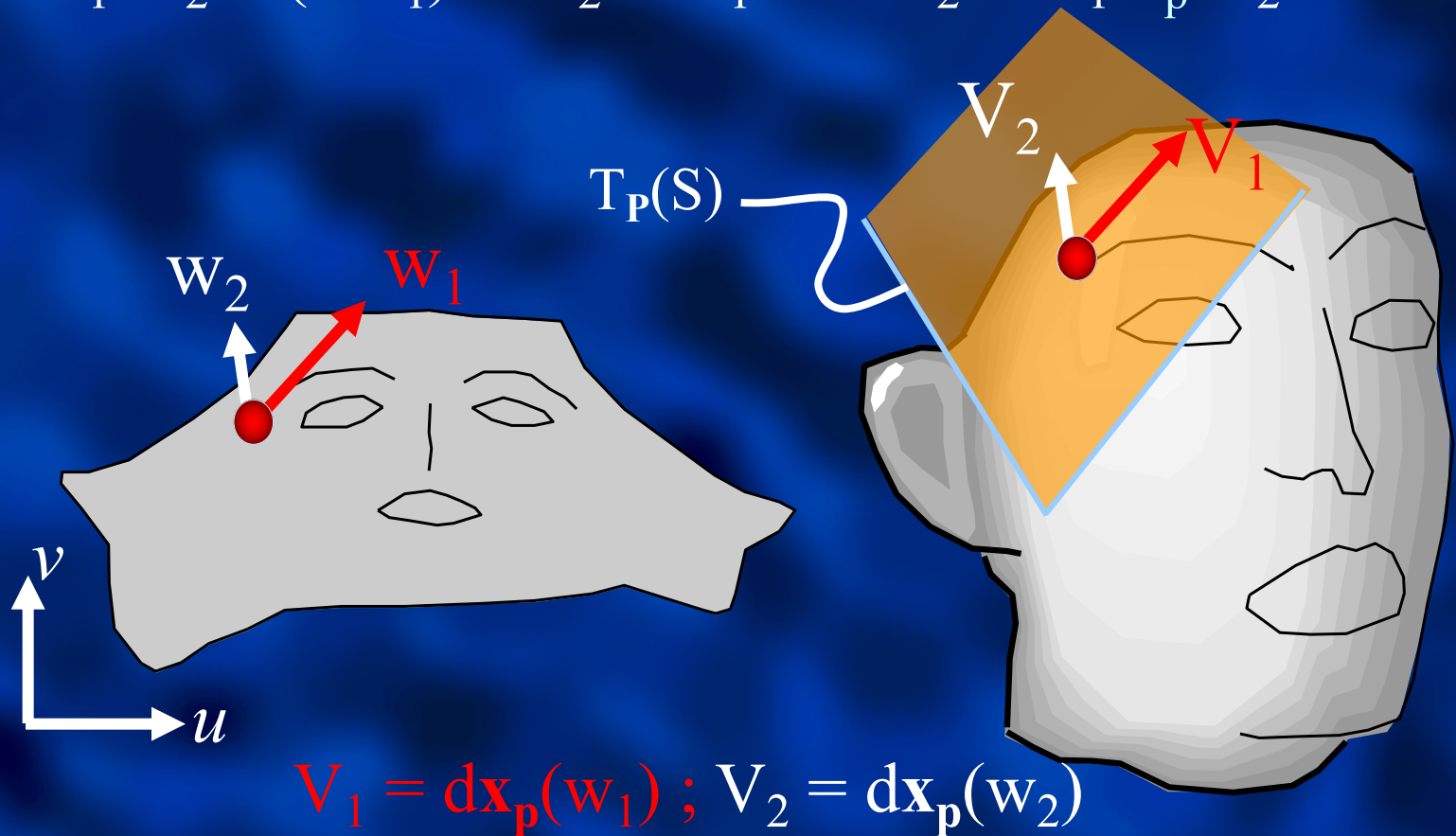
$$d\mathbf{x}_P(\mathbf{w}) = \mathbf{w}_u \partial x / \partial u + \mathbf{w}_v \partial x / \partial v = J_P \cdot \mathbf{w}$$



Geometry of $T_p(S)$

Measuring things, First Fundamental Form I_p

$$V_1^t V_2 = (J w_1)^t J w_2 = w_1^t J^t J w_2 = w_1^t I_p w_2$$



Geometry of $T_p(S)$

Measuring things, First Fundamental Form I_p

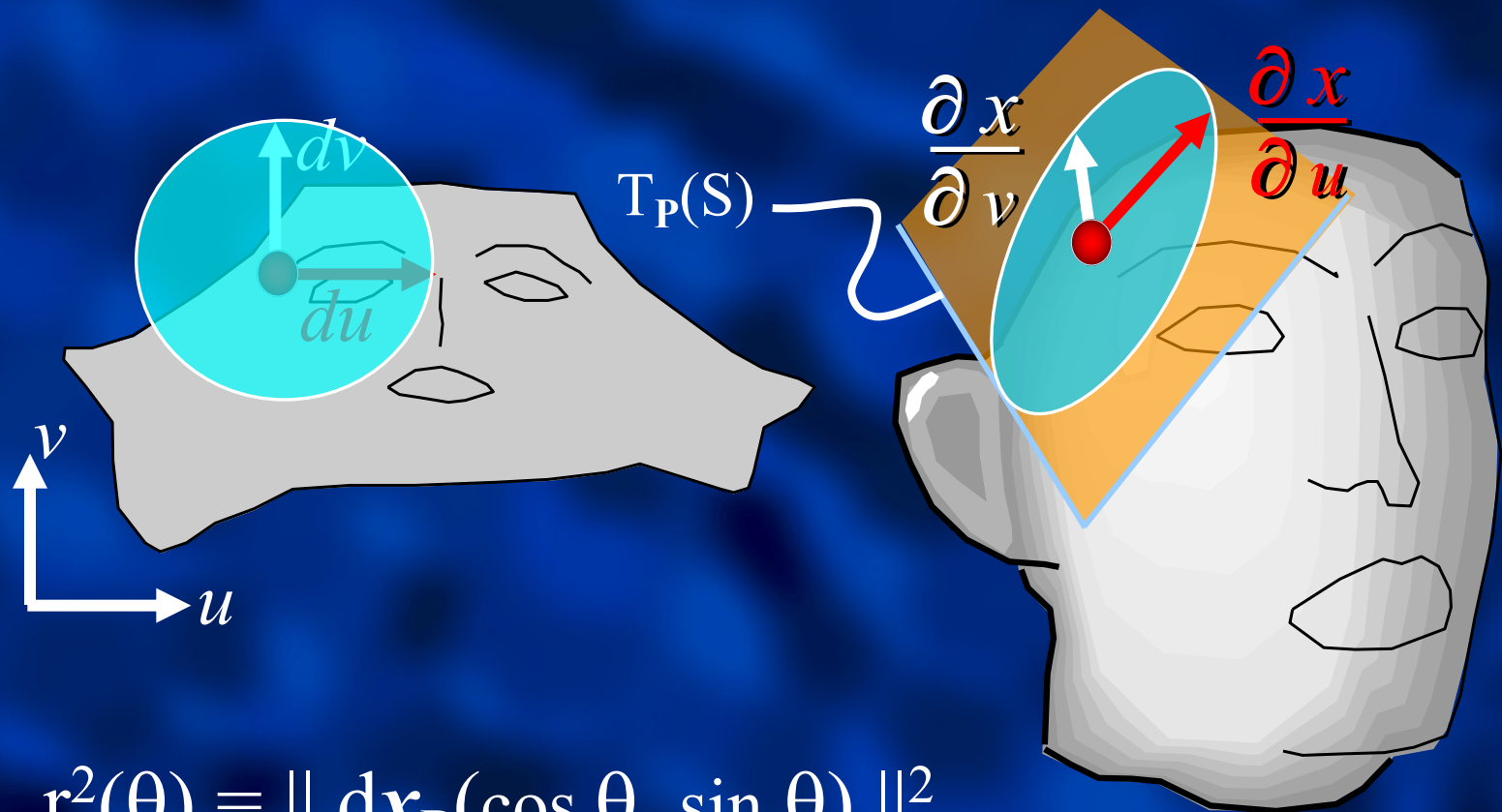
$$\text{Distances : } \| V_1 \|^2 = w_1^t I_p w_1$$

$$\text{Angles : } V_1^t V_2 = w_1^t I_p w_2$$

I_p is called the *metric tensor*

Geometry of $T_p(S)$

Anisotropy



$$r^2(\theta) = \| d\mathbf{x}_p(\cos \theta, \sin \theta) \|^2$$

Geometry of $T_p(S)$

Anisotropy ; 1st fundamental form I_p

$$\| d\mathbf{x}_p(\mathbf{w}) \|^2 = \| \mathbf{J}_p \cdot \mathbf{w} \|^2$$

$$= (\mathbf{J}_p \mathbf{w}) \cdot (\mathbf{J}_p \mathbf{w})^t$$

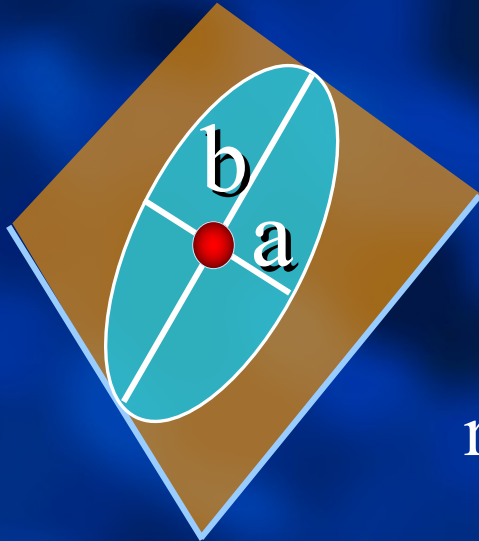
$$= \mathbf{w}^t \cdot \mathbf{J}_p^t \cdot \mathbf{J}_p \cdot \mathbf{w}$$

$$= \mathbf{w}^t \cdot \mathbf{I}_p \cdot \mathbf{w}$$

$$\mathbf{I}_p = \begin{bmatrix} \frac{\partial \mathbf{x}}{\partial u} \cdot \frac{\partial \mathbf{x}}{\partial u} & \frac{\partial \mathbf{x}}{\partial u} \cdot \frac{\partial \mathbf{x}}{\partial v} \\ \frac{\partial \mathbf{x}}{\partial u} \cdot \frac{\partial \mathbf{x}}{\partial v} & \frac{\partial \mathbf{x}}{\partial v} \cdot \frac{\partial \mathbf{x}}{\partial v} \end{bmatrix}$$

Geometry of $T_p(S)$

Anisotropy ; 1st fundamental form I_p



$\mathbf{w}_1, \mathbf{w}_2$ unit eigen vectors of I_p

λ_1, λ_2 associated eigen values

$$r^2(\theta) = \| d\mathbf{x}_p(\cos \theta \mathbf{w}_1 + \sin \theta \mathbf{w}_2) \|^2$$

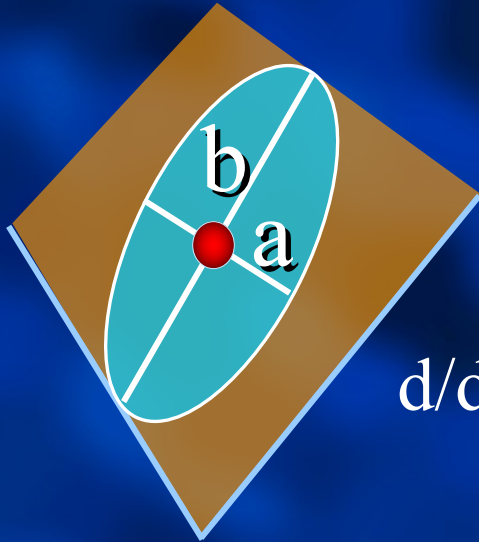
$$= (\cos \theta \cdot \mathbf{w}_1 + \sin \theta \cdot \mathbf{w}_2)^t \cdot \mathbf{I}_p \cdot (\cos \theta \cdot \mathbf{w}_1 + \sin \theta \cdot \mathbf{w}_2)$$

$$= \cos^2 \theta \cdot \|\mathbf{w}_1\|^2 \cdot \lambda_1 + \sin^2 \theta \cdot \|\mathbf{w}_2\|^2 \cdot \lambda_2 + \\ \sin \theta \cdot \cos \theta \cdot (\lambda_1 \cdot \mathbf{w}_2^t \cdot \mathbf{w}_1 + \lambda_2 \cdot \mathbf{w}_1^t \cdot \mathbf{w}_2)$$

$$r^2(\theta) = \cos^2 \theta \cdot \lambda_1 + \sin^2 \theta \cdot \lambda_2$$

Geometry of $T_p(S)$

Anisotropy ; eigen structure of I_p



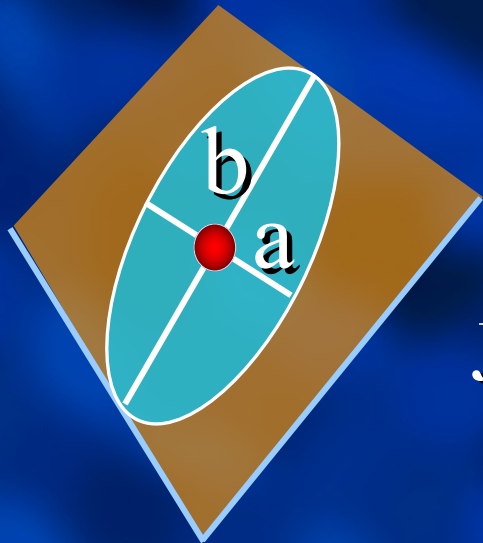
$$r^2(\theta) = \cos^2 \theta \cdot \lambda_1 + \sin^2 \theta \cdot \lambda_2$$

$$\begin{aligned} \frac{d}{d\theta} (r^2(\theta)) &= 2 \cdot \sin \theta \cdot \cos \theta \cdot (\lambda_2 - \lambda_1) \\ &= \sin(2\theta) \cdot (\lambda_2 - \lambda_1) \end{aligned}$$

$$a = \sqrt{\lambda_1} \quad ; \quad b = \sqrt{\lambda_2} \quad (\text{eigen values of } I_p)$$

Geometry of $T_p(S)$

Anisotropy ; eigen structure of I_p



$$J_p = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} \end{bmatrix} = U \begin{bmatrix} a & 0 \\ 0 & b \\ 0 & 0 \end{bmatrix} V^t$$

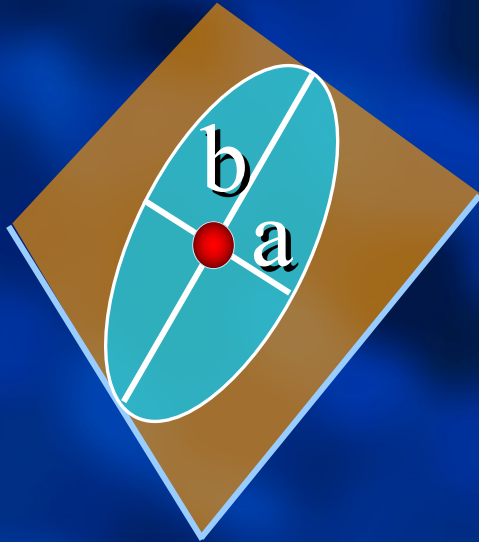
See also
[Hormann]

Singular values decomposition (SVD) of J

$$\text{Rem: } I_p = J^t \cdot J \implies a = \sqrt{\lambda_1} \quad ; \quad b = \sqrt{\lambda_2}$$

2. Simple Objects

Anisotropy ; axes

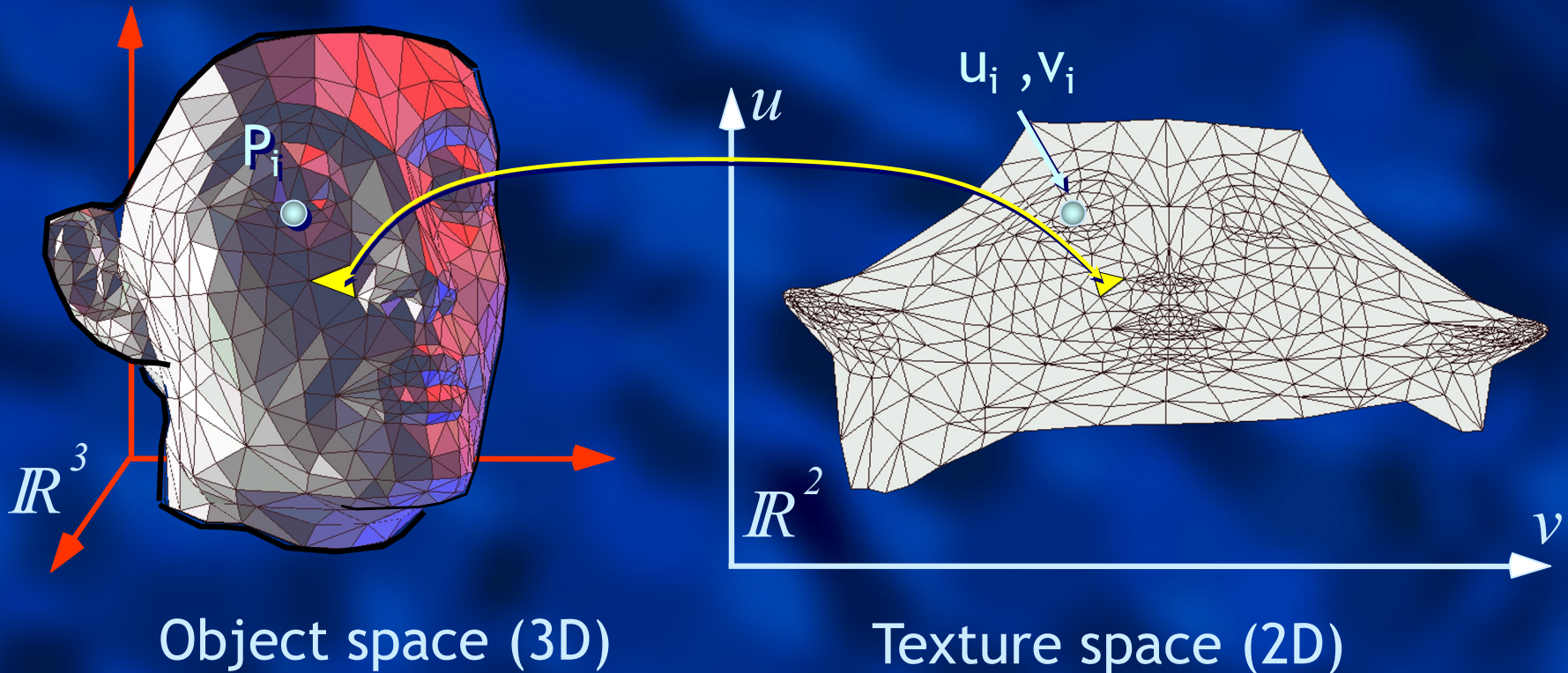


$$\mathbf{I}_p = \begin{bmatrix} \frac{\partial \mathbf{x}}{\partial u}^2 & \frac{\partial \mathbf{x}}{\partial u} \cdot \frac{\partial \mathbf{x}}{\partial v} \\ \frac{\partial \mathbf{x}}{\partial u} \cdot \frac{\partial \mathbf{x}}{\partial v} & \frac{\partial \mathbf{x}}{\partial v}^2 \end{bmatrix}$$

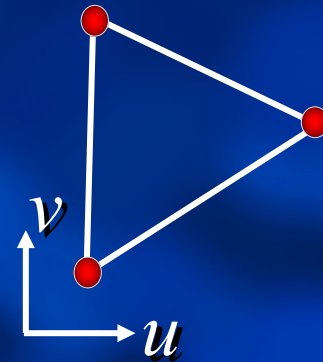
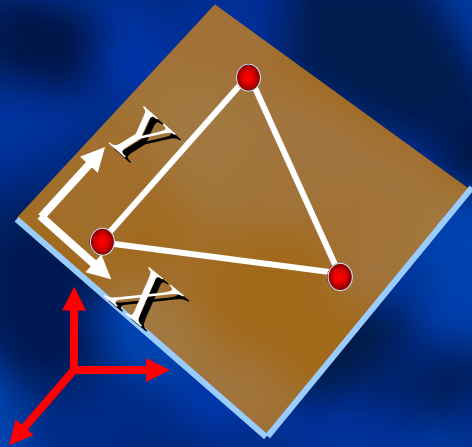
$$a = \sqrt{\lambda_1} \quad ; \quad b = \sqrt{\lambda_2} \quad (\text{eigen values of } \mathbf{I}_p = \text{sq. sing values of } \mathbf{J}_p)$$

2. Simple Objects

Triangulated surfaces



2. Using Differential Geometry *Triangulated Surfaces*



$$\begin{aligned} J_T &= \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{pmatrix} \\ &= \frac{1}{2|T|_{x,y}} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_3 - x_2 & x_1 - x_3 & x_2 - x_1 \\ y_3 - y_2 & y_1 - y_3 & y_2 - y_1 \end{pmatrix} \begin{pmatrix} u_1 & v_1 \\ u_2 & v_2 \\ u_3 & v_3 \end{pmatrix} \end{aligned}$$

2. Using Differential Geometry

*Anisotropy - See SIGGRAPH & EUROGRAPHICS
course notes (with Kai Hormann and Alla Sheffer)*

- first fundamental form $\mathbf{I}_f = J_f^T J_f = \begin{pmatrix} E & F \\ F & G \end{pmatrix}$

$$E = f_u^T f_u \quad F = f_u^T f_v \quad G = f_v^T f_v$$

- eigenvalues of $\mathbf{I}_f$

$$\lambda_{1,2} = \frac{1}{2} \left((E + G) \pm \sqrt{4F^2 + (E - G)^2} \right)$$

- singular values of J_f (anisotropy ellipse axes)

$$\sigma_1 = \sqrt{\lambda_1} \quad \sigma_2 = \sqrt{\lambda_2}$$

Non-Distorted Parameterization

General Idea

- Many different applications (with different requirements)
- A nice formalism (differential geometry)
- Lots of different methods

Non-Distorted Parameterization

General Idea

- Define some energy functional F in function of $J_p, I_p, \lambda_1, \lambda_2$
- Expand their expression in F in function of the unknown u_i, v_i
- Design an algorithm to find the u_i, v_i 's that minimizes F

2. Simple Objects

Some Parameterization Methods

$$\|\mathbf{I} - \mathbf{Id}\|_F^2 = (\lambda_1 - 1)^2 + (\lambda_2 - 1)^2$$

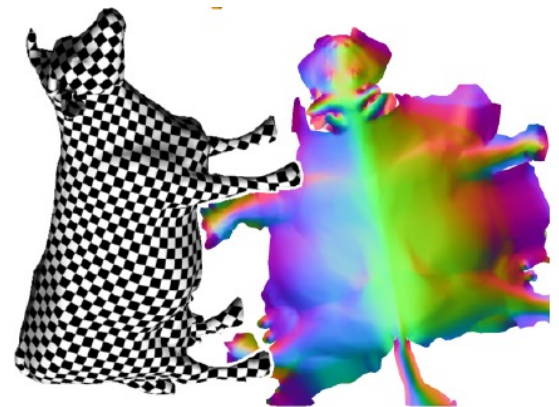
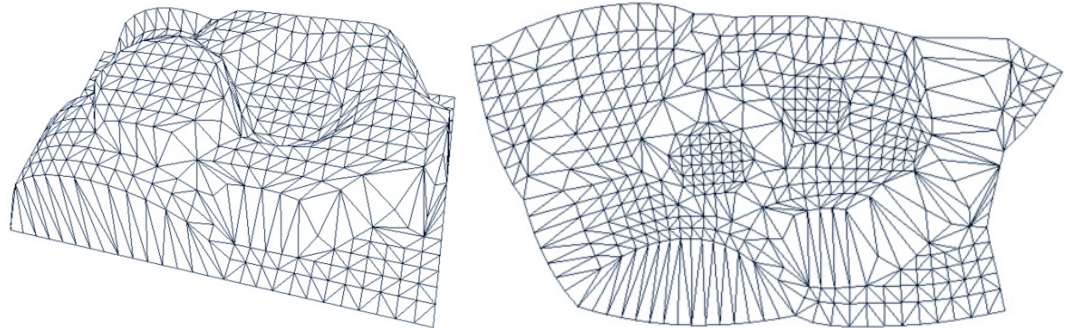
[Maillot & Verroust]

$$\frac{\sigma_1^2 + \sigma_2^2}{\sigma_1 \sigma_2} = \frac{\text{trace}(\mathbf{I}_T)}{\det(\mathbf{J}_T)}$$

[Hormann & Greiner]

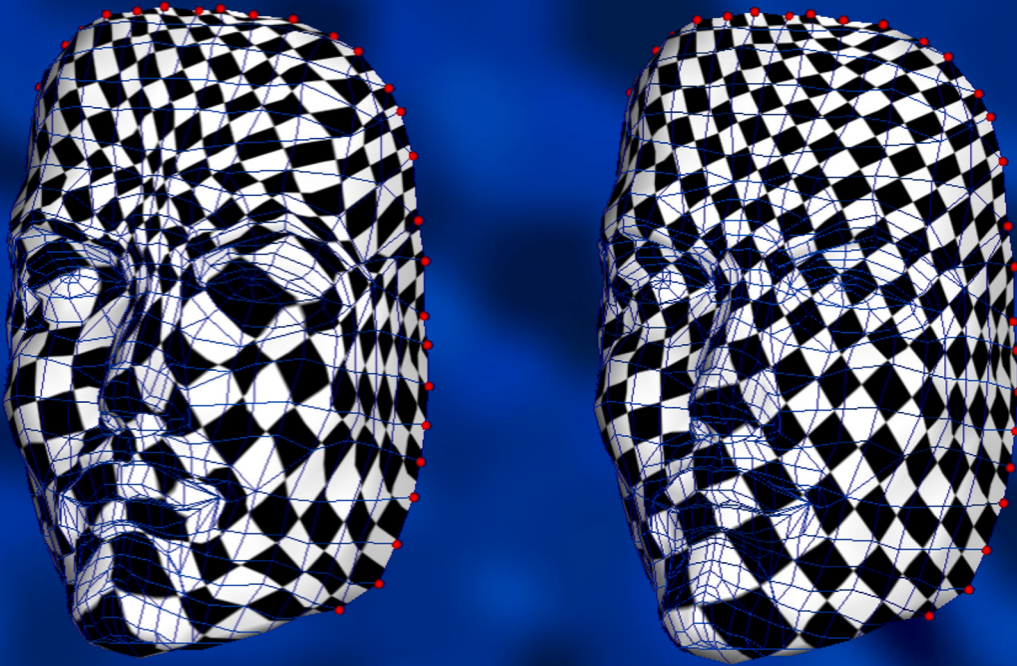
$$\begin{aligned} L_2(T) &= \sqrt{((1/\sigma_1)^2 + (1/\sigma_2)^2) / 2} \\ L_\infty(T) &= 1/\sigma_1 \end{aligned}$$

[Sander & Hoppe]



2. Simple Objects

Mean Value Coordinates



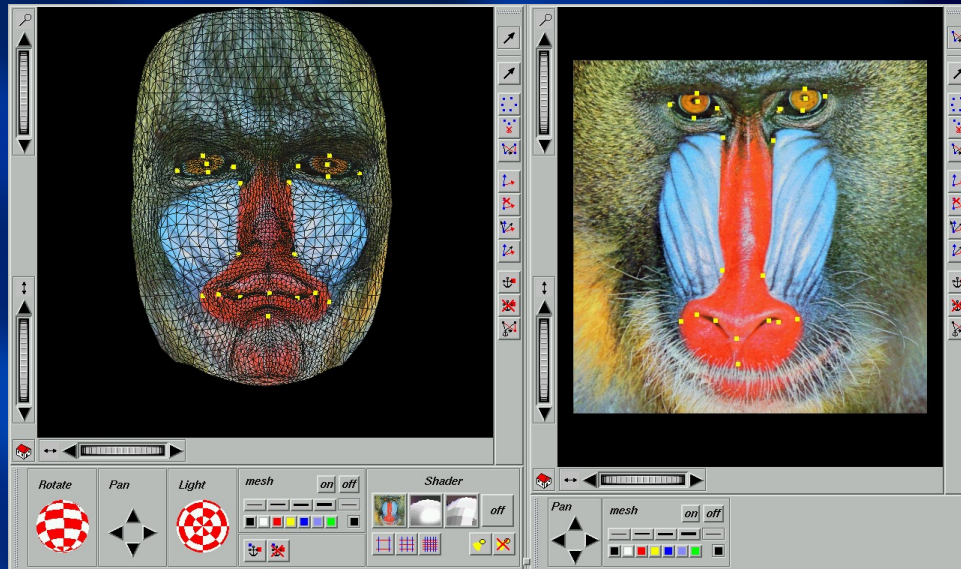
$$a_{ij} = 1$$

[Floater] mean value coordinates

The only one with th. guarantees

$$\frac{\tan \frac{\alpha_{ij}}{2} + \tan \frac{\beta_{ji}}{2}}{r_{ij}}$$

2. Simple Objects *Constraints*



DEMO

[Levy 01]



Interactive, no guarantee

MatchMaker [Kraevoy et.al]

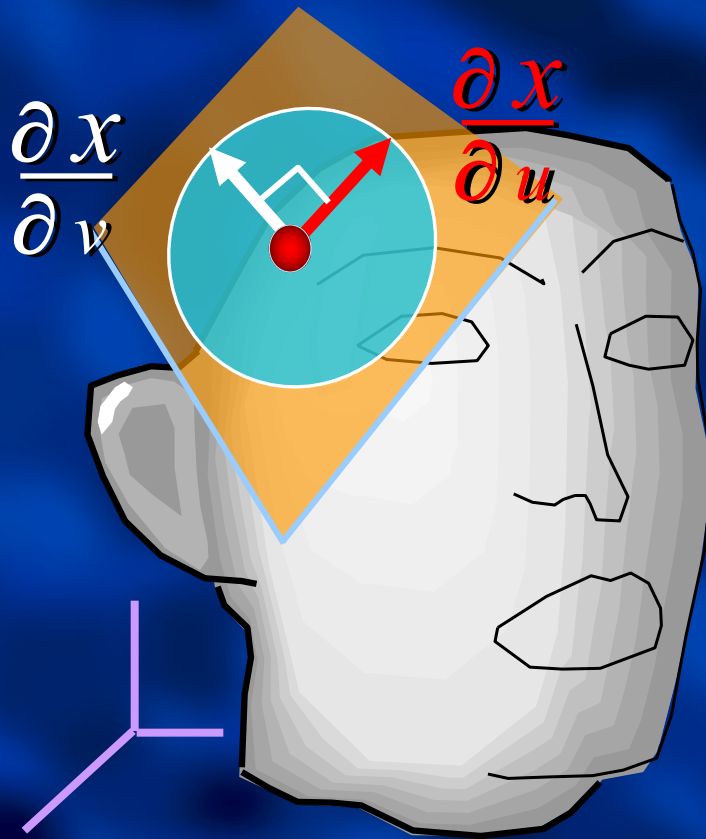


TextureMontage [Zhou et.al]

Non-interactice, 1 to 1

2. Simple Objects

Conformal Parameterization

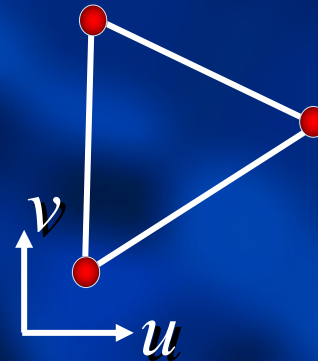
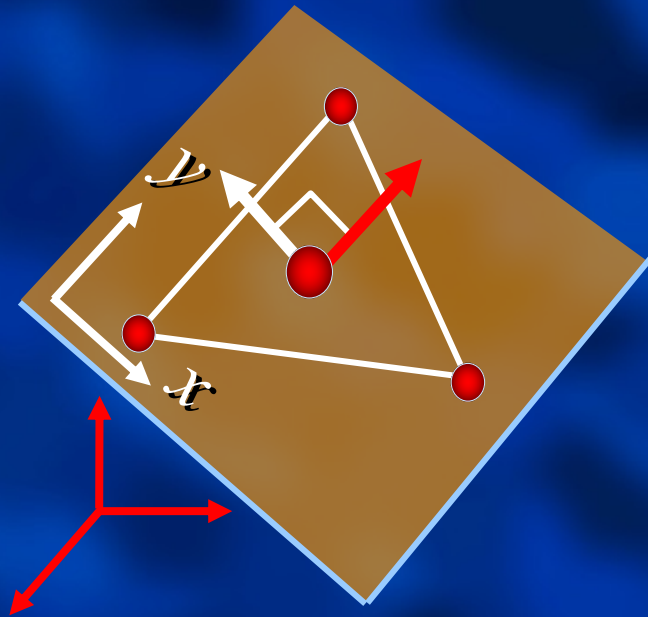


$$\lambda_2 = \lambda_1$$

$$\frac{\partial x}{\partial v} \wedge N = \frac{\partial x}{\partial u}$$

2. Simple Objects

Conformal Parameterization



Cauchy-Riemann:

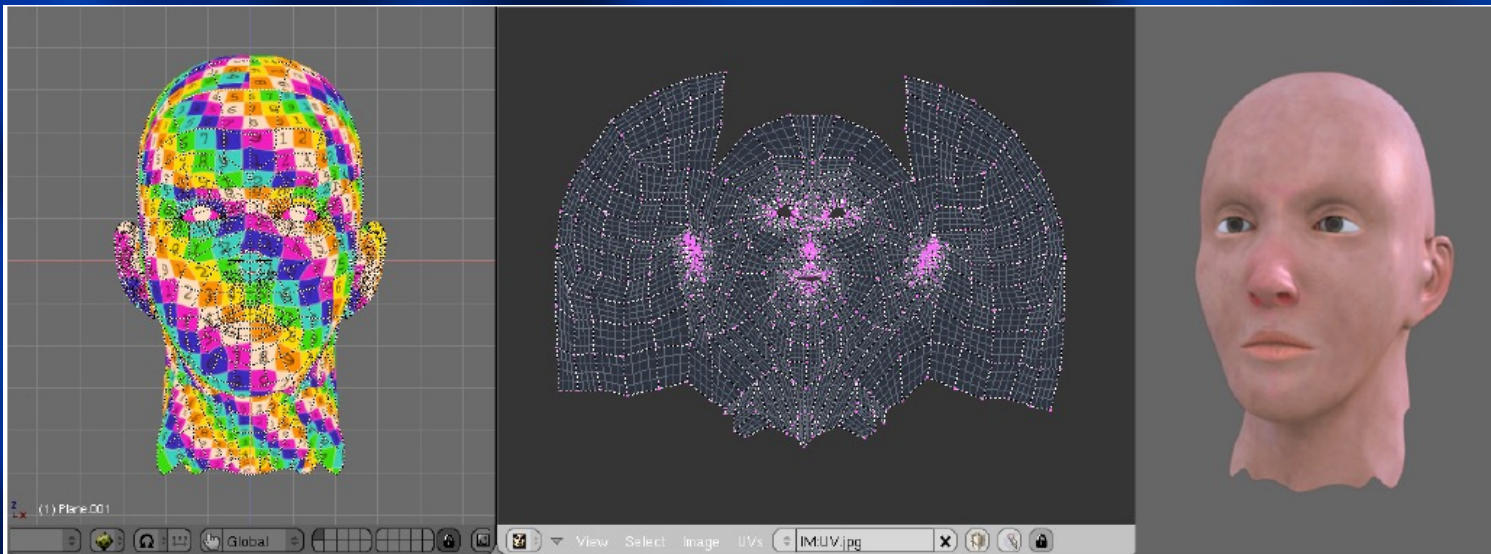
$$\begin{cases} \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} \end{cases}$$

No Piecewise Linear solution in general

2. Simple Objects

LSCM

Minimize $\sum_T \left\| \begin{bmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \end{bmatrix} - \begin{bmatrix} \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial x} \end{bmatrix} \right\|^2$ Fix two vertices to determine rot,transl,scaling



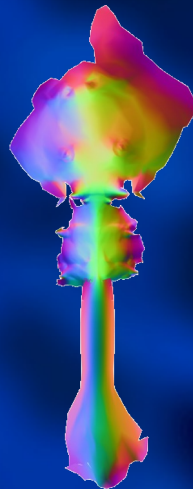
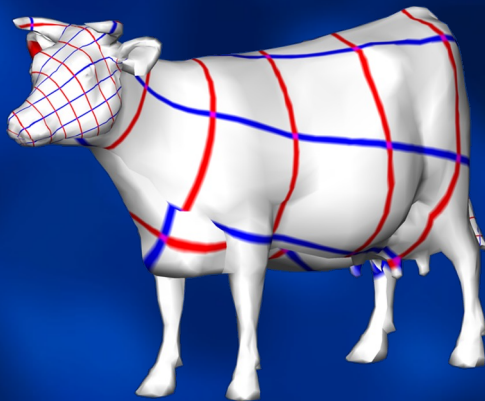
+ easy to implement - overlaps, deformations

2. Simple Objects

ABF/ABF++/LinABF

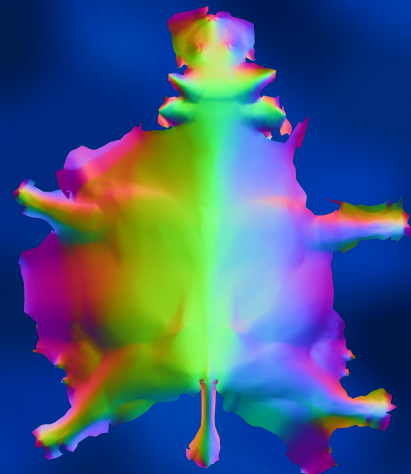
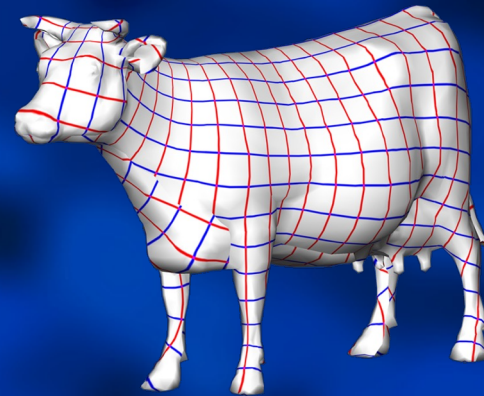
Limits of analytic methods

LSCM ; DNCP



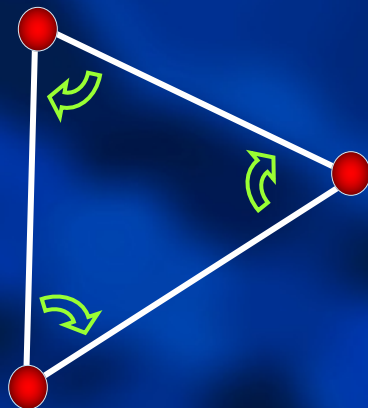
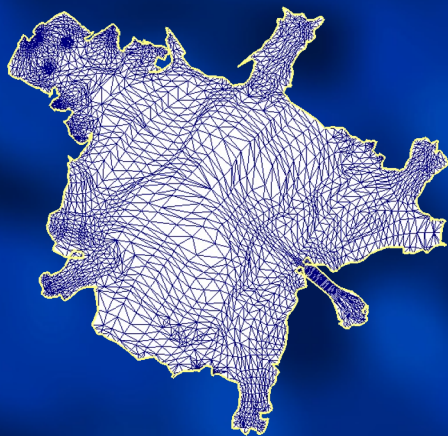
distortions ; validity

Geometric methods →



2. Simple Objects

ABF/ABF++/LinABF



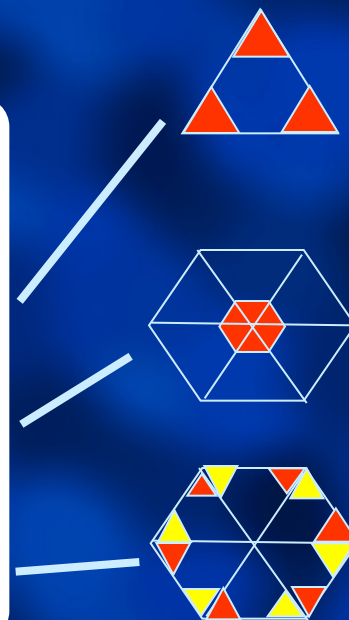
Variables: angles

characterization of planar triangulation

$$F(x) = F \left(\begin{array}{c} \alpha \\ \lambda \end{array} \right) = \sum_{k=1}^{3.nf} \frac{1}{w_k} (\alpha_k - \beta_k)^2 + \sum_{k=1}^{nf+2.nv} \lambda_k \cdot C_k(\alpha)$$

where:

$$\left\{ \begin{array}{ll} C_{1...nf} & = \alpha_i + \alpha_j + \alpha_k - \pi \\ C_{nf+1...nf+nv} & = \sum \alpha_k - 2.\pi \\ C_{nf+nv...nf+2.nv} & = \prod \sin(\alpha^+) - \prod \sin(\alpha^-) \end{array} \right.$$

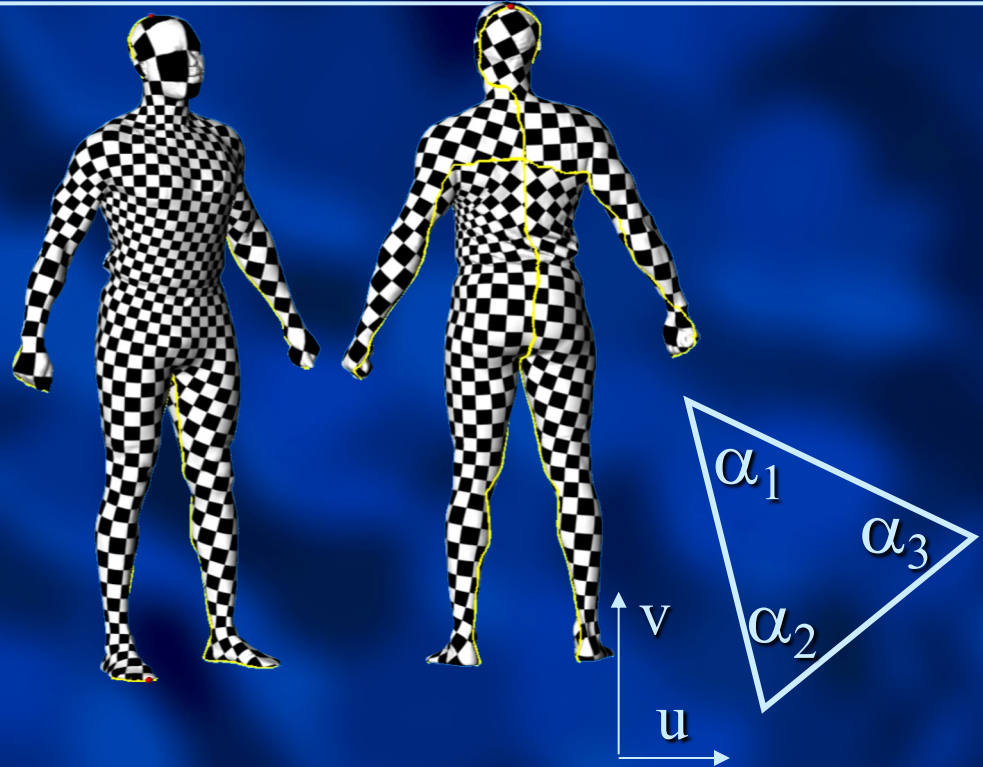


2. Simple Objects

ABF/ABF++/LinABF

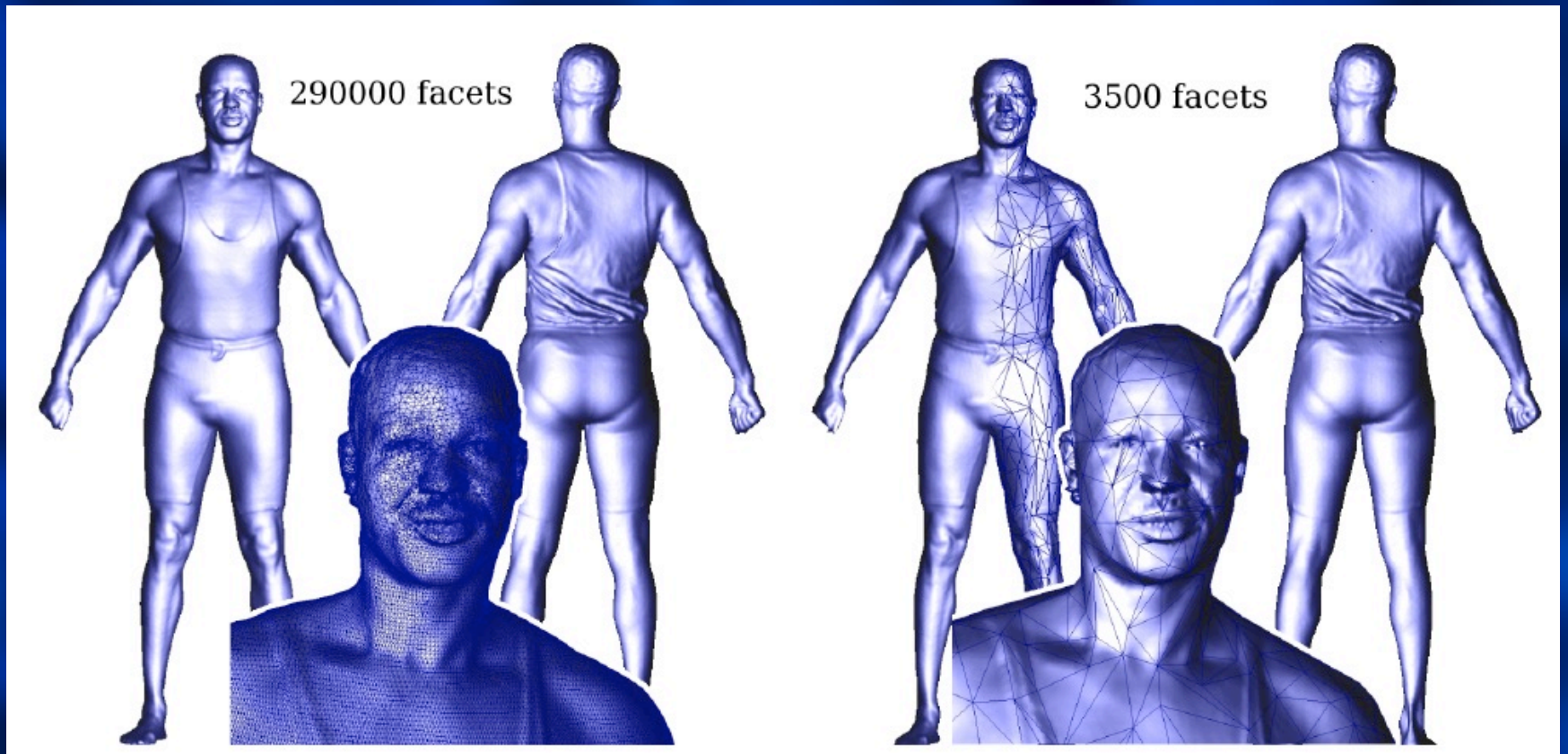


Cut by Seamster
[Sheffer & Hart]



initial ABF [Sheffer & de Sturler]
faster one [Sheffer et.al 04]
linABF [Zayer et.al 07]

2. Simple Objects



Demo: Normal mapping

2. Simple objects

Summary

■ **Fixed boundary**

Floater MVC: provably correct

■ **Free boundary**

LSCM: + linear, easy to implement

- trgl flip, distortions

(lin)ABF(++): + lower distortion

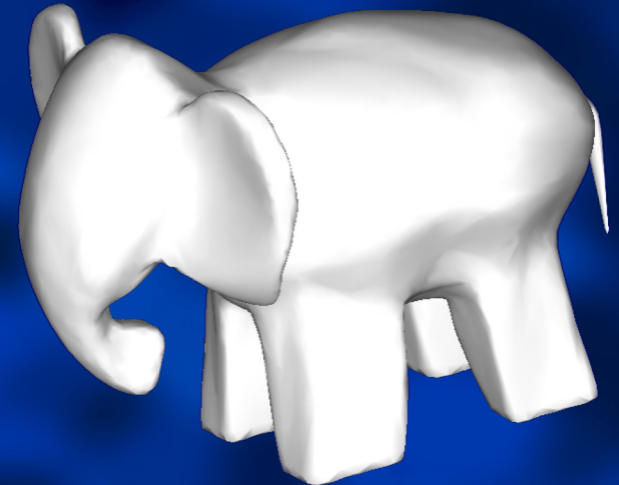
- more complicated

[Zayer07] LinABF: ABF in 30 lines of Matlab !!

(C++ sources soon on <http://alice.loria.fr>)

3. More complicated objects

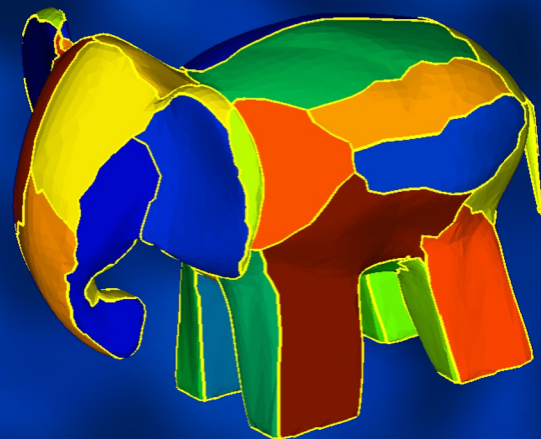
- If the object is not unfoldable



... we need to cut it.

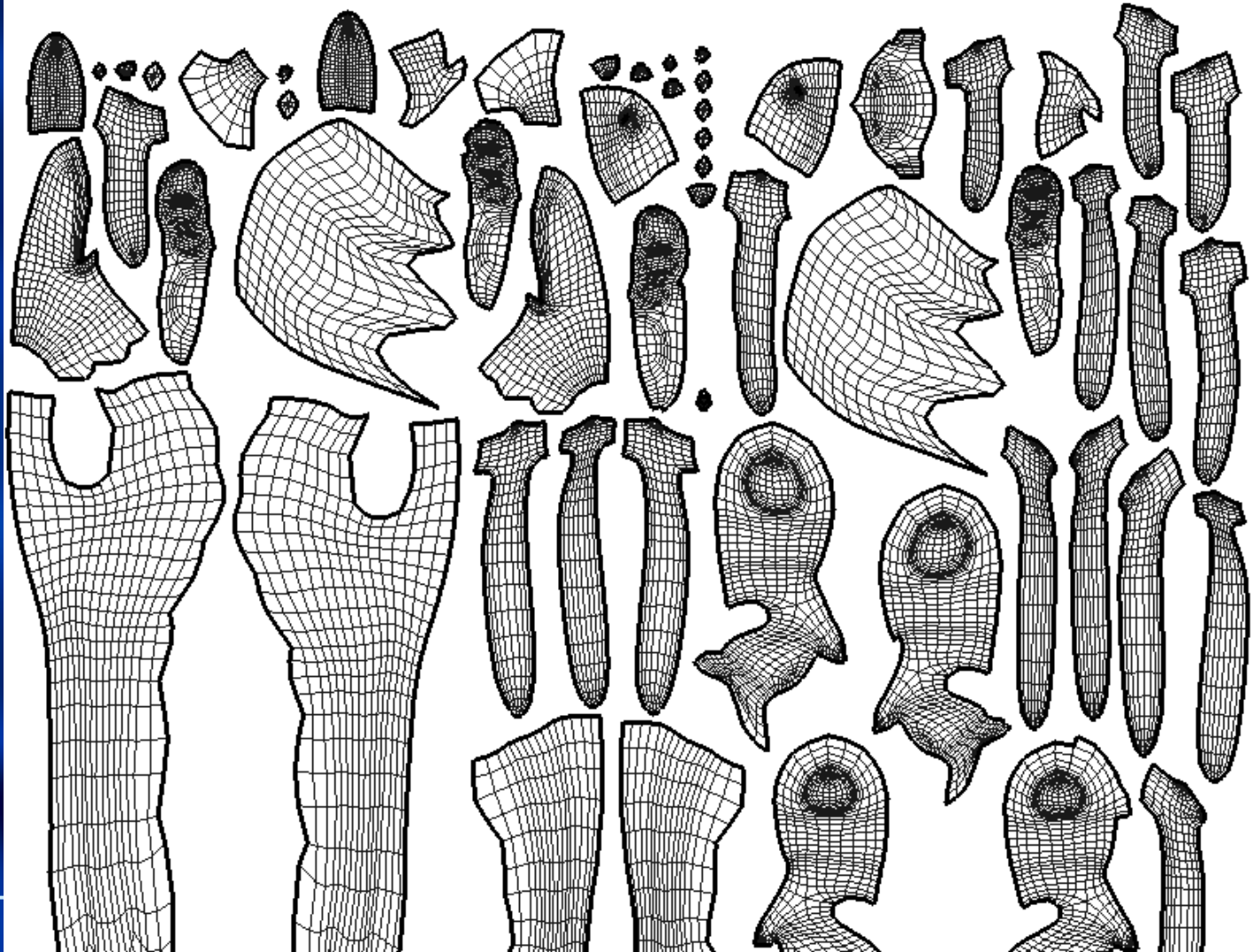
VSA [Alliez et.al 04]

DEMO



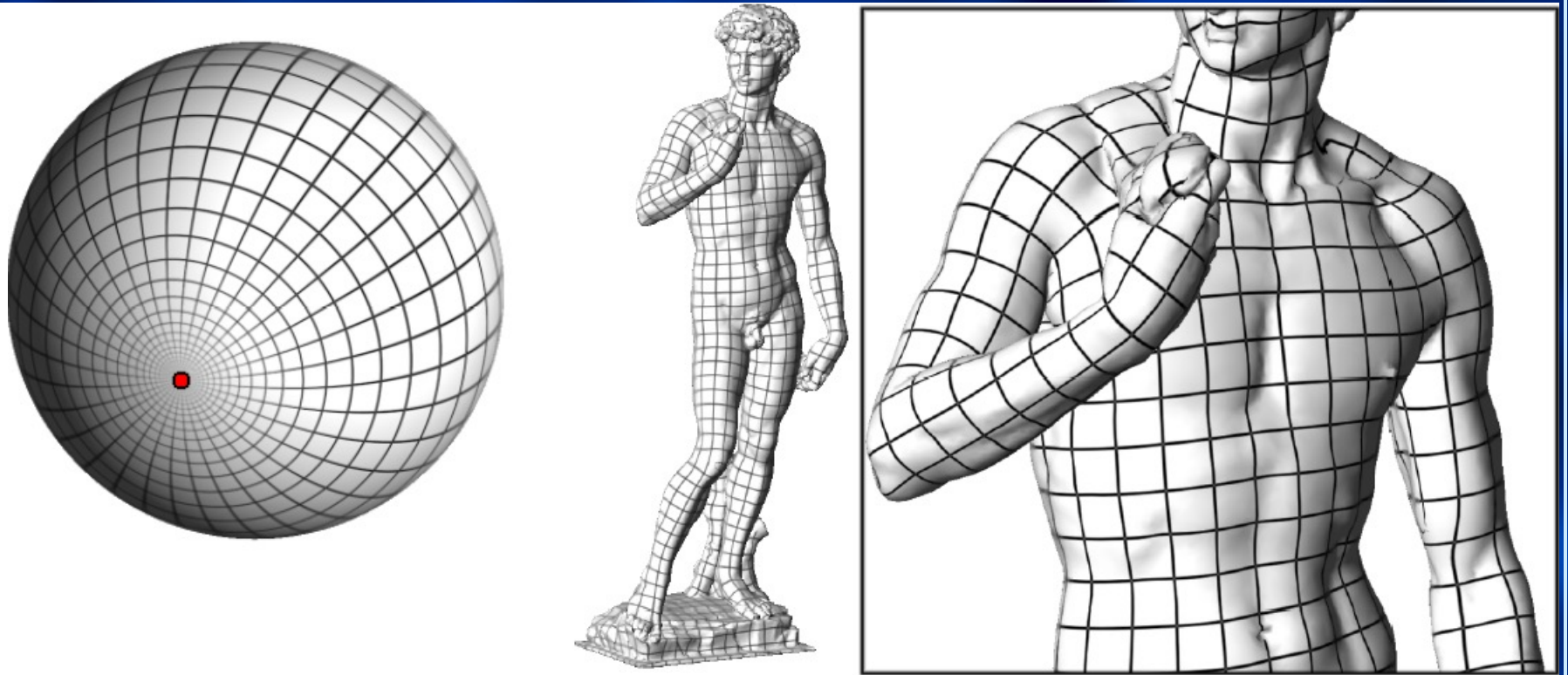
3. More complicated objects

Ray's "Tetris" packing (DXUVAtlas)



3. More Complicated Objects

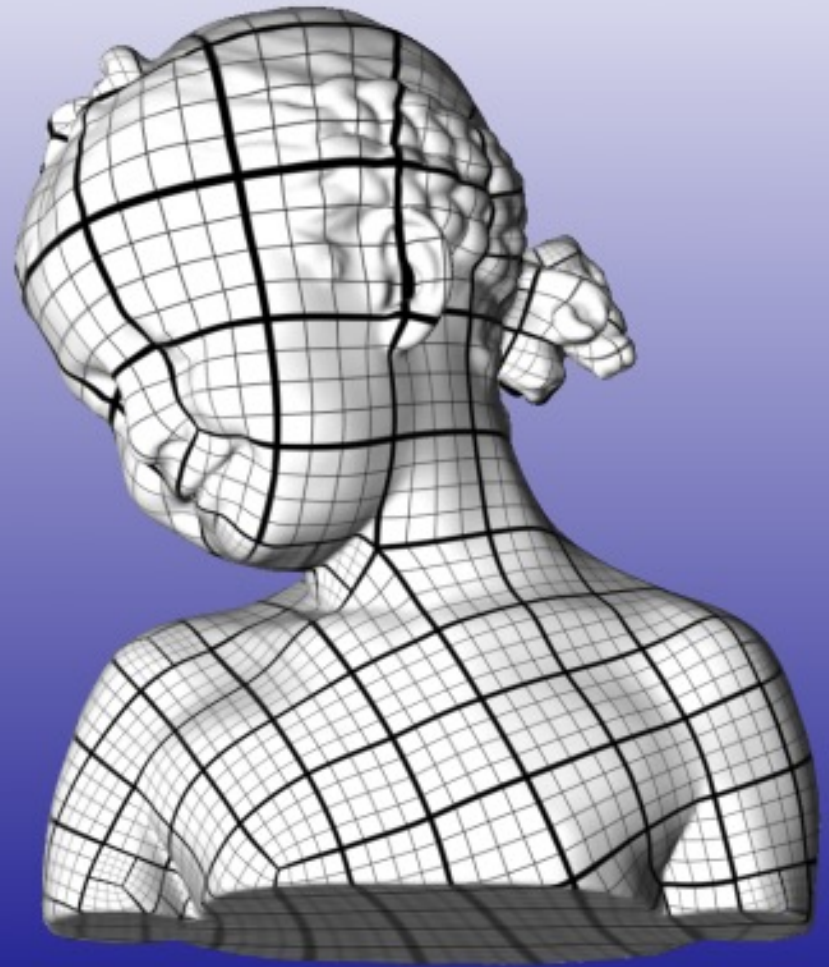
Arbitrary Topology : Need for new methods



Create a « geographic coordinate system »

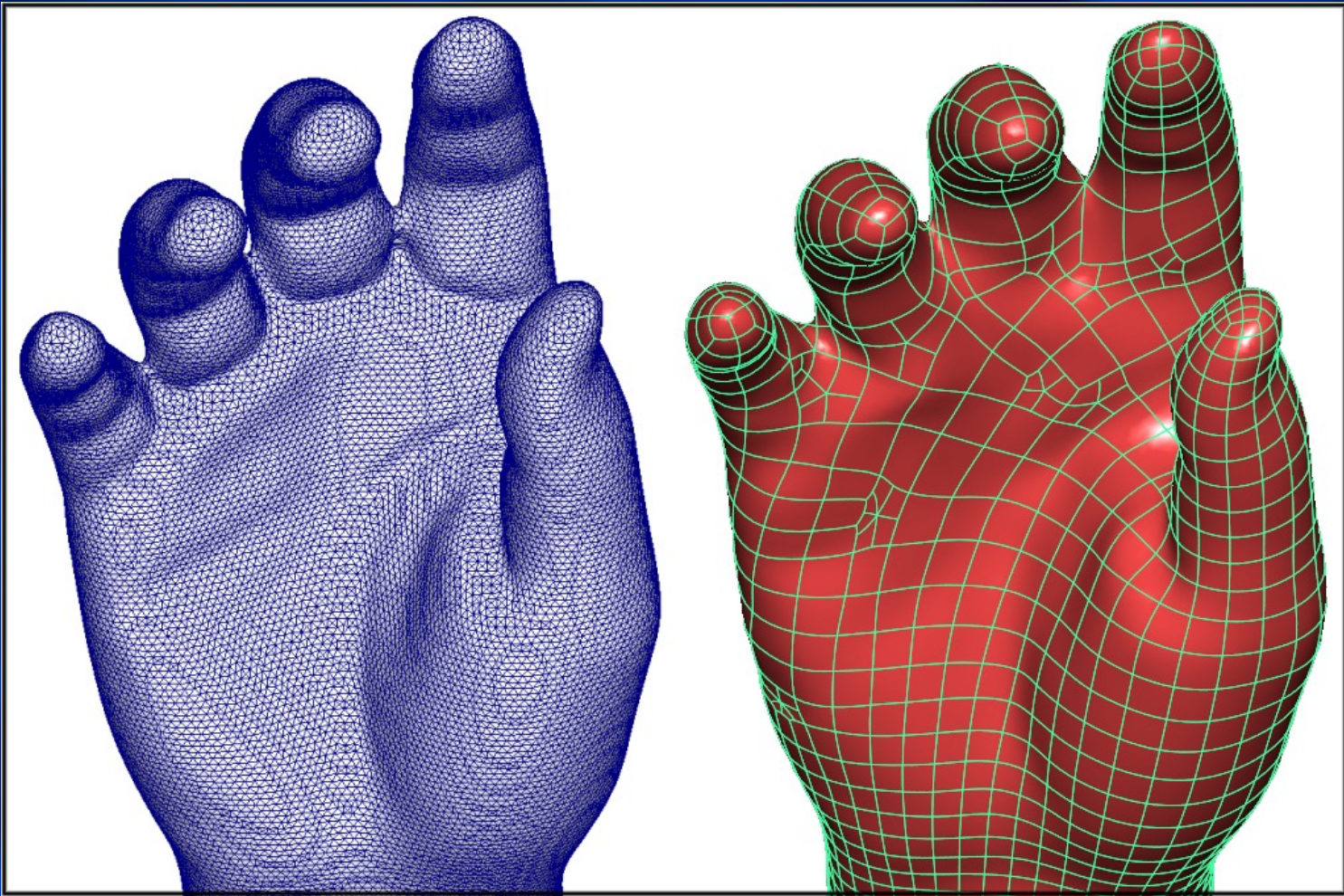
3. More Complicated Objects

Remeshing and Geometry Images



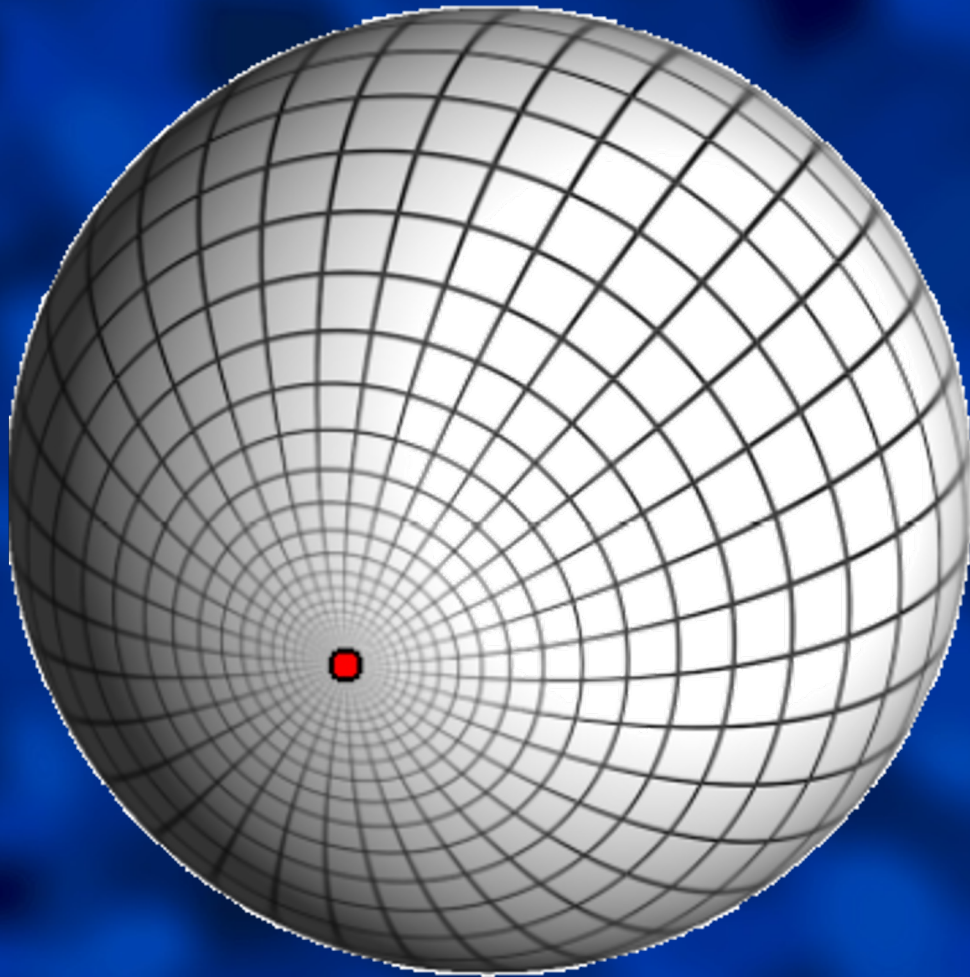
3. More Complicated Objects

Reverse engineering



3. Difficulties

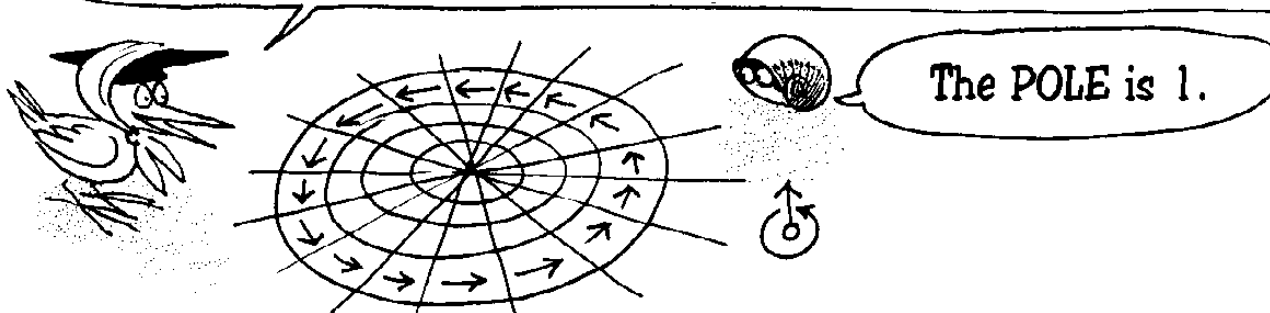
Singularities



3. Difficulties

Singularities

THE ORDER OF SINGULARITY OF A WEAVE is equal to the angle of the arrow's direction, positive or negative, divided by 360° (2π)

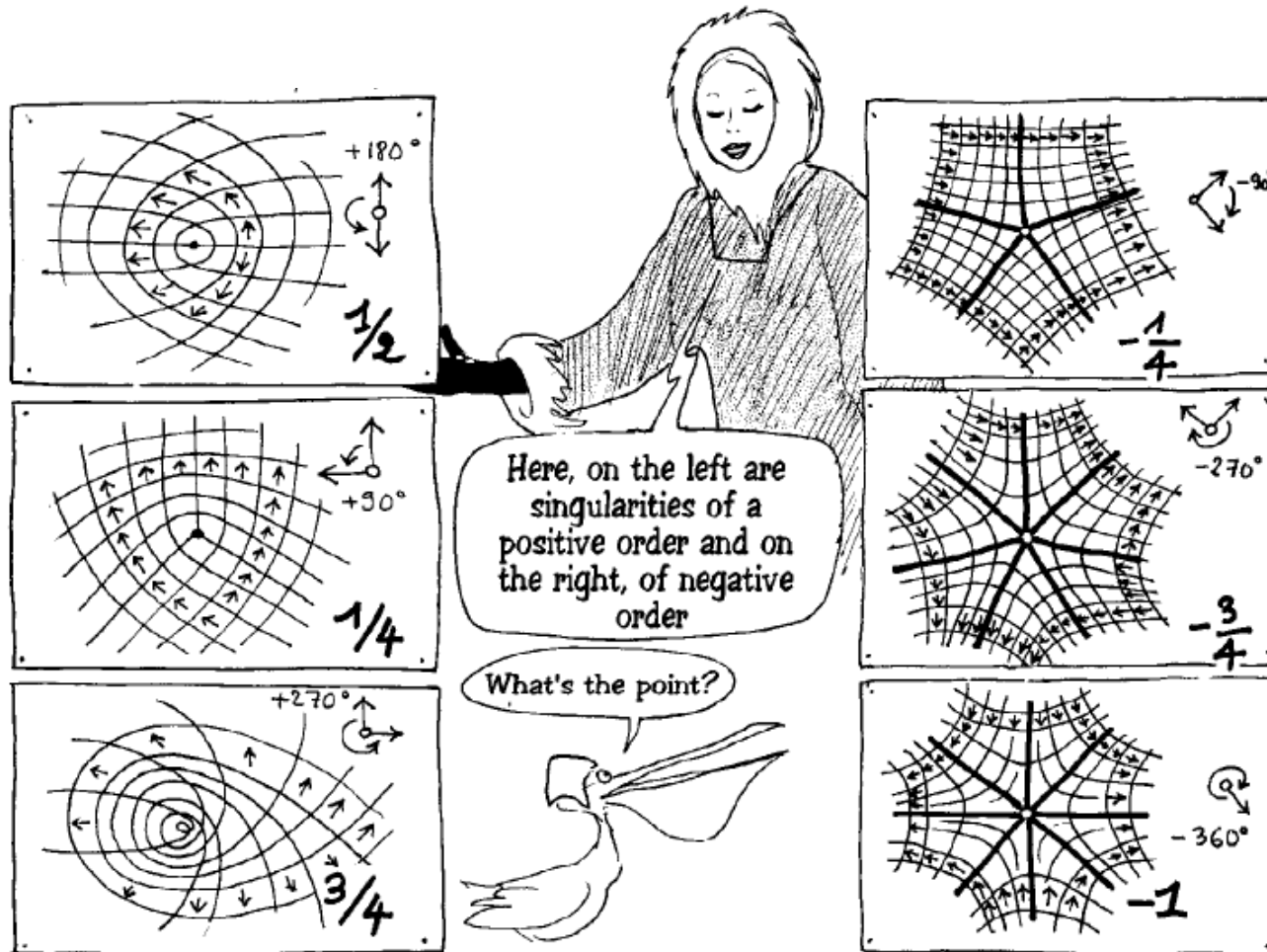


Archibald Higgins Adventures (used with author permission)
For french readers, Archibald is known as Anselme Lanturlu

<http://www.savoir-sans-frontieres.org>

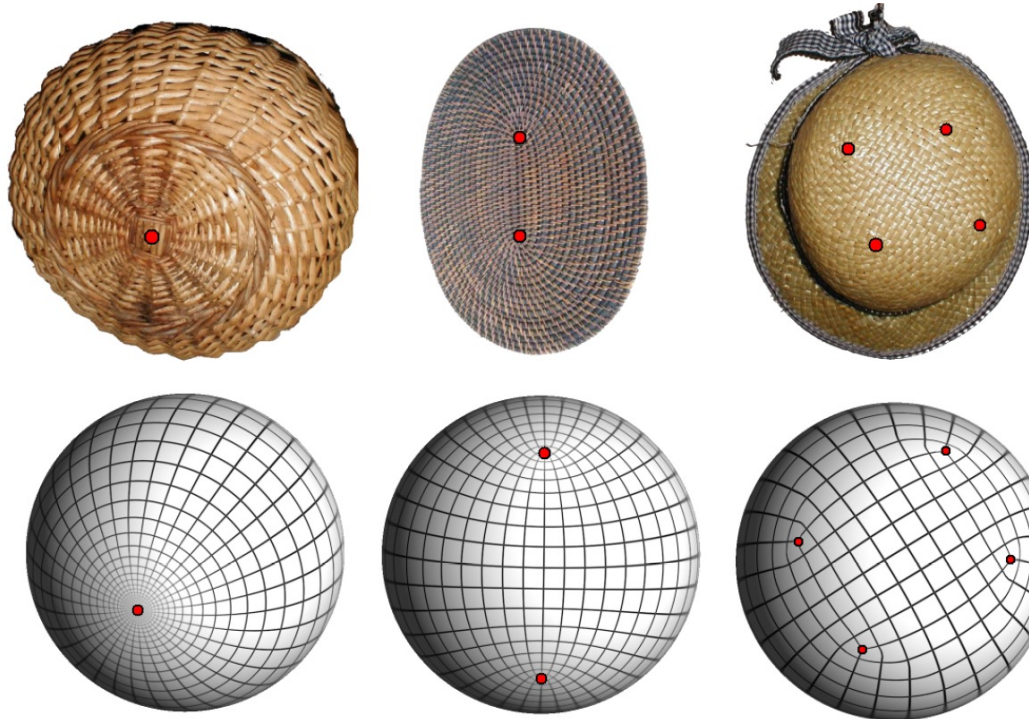
3. Difficulties

Singularities

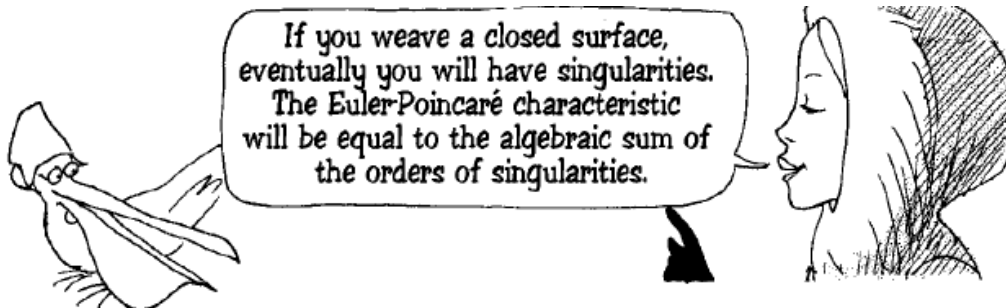


3. Difficulties

Singularities



If you weave a closed surface,
eventually you will have singularities.
The Euler-Poincaré characteristic
will be equal to the algebraic sum of
the orders of singularities.



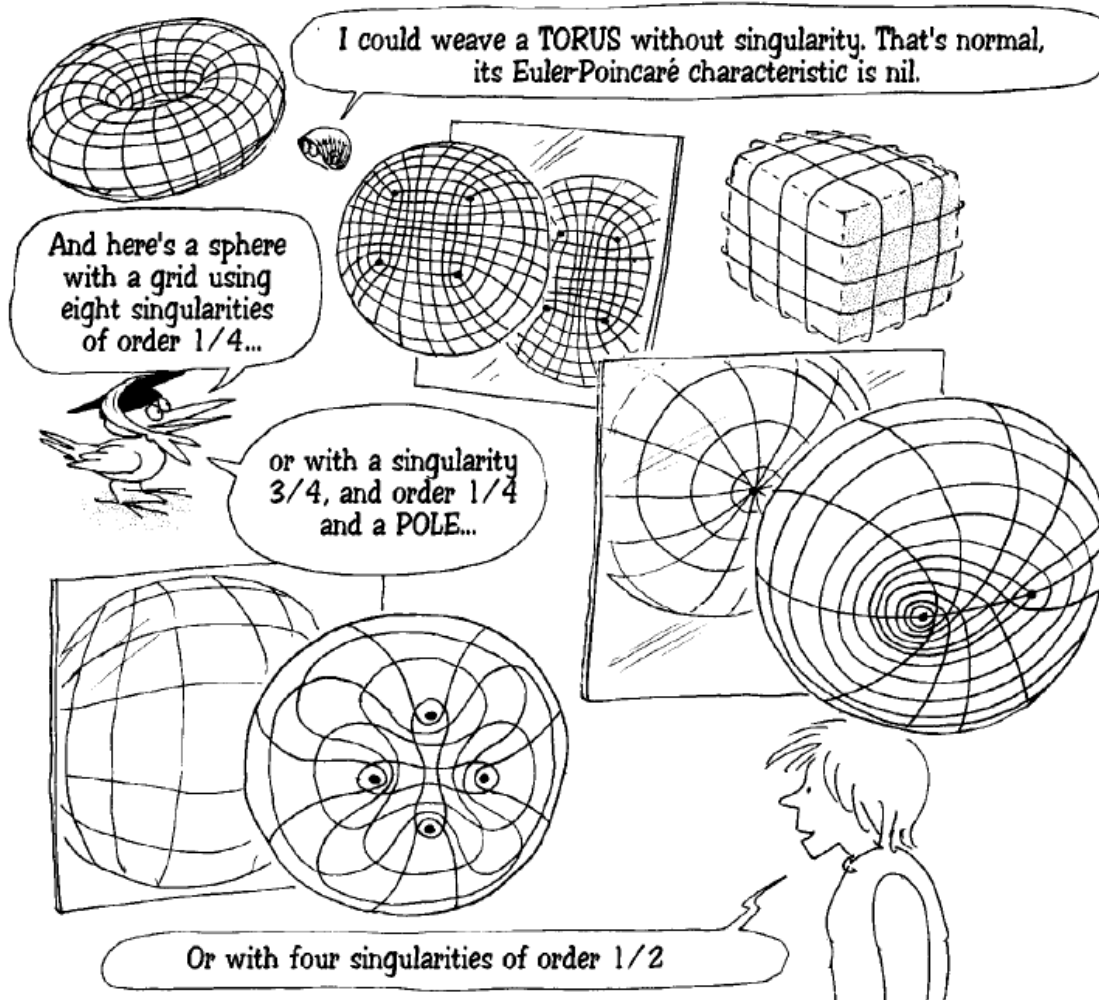
3. Difficulties

Singularities

Poincaré Hopf

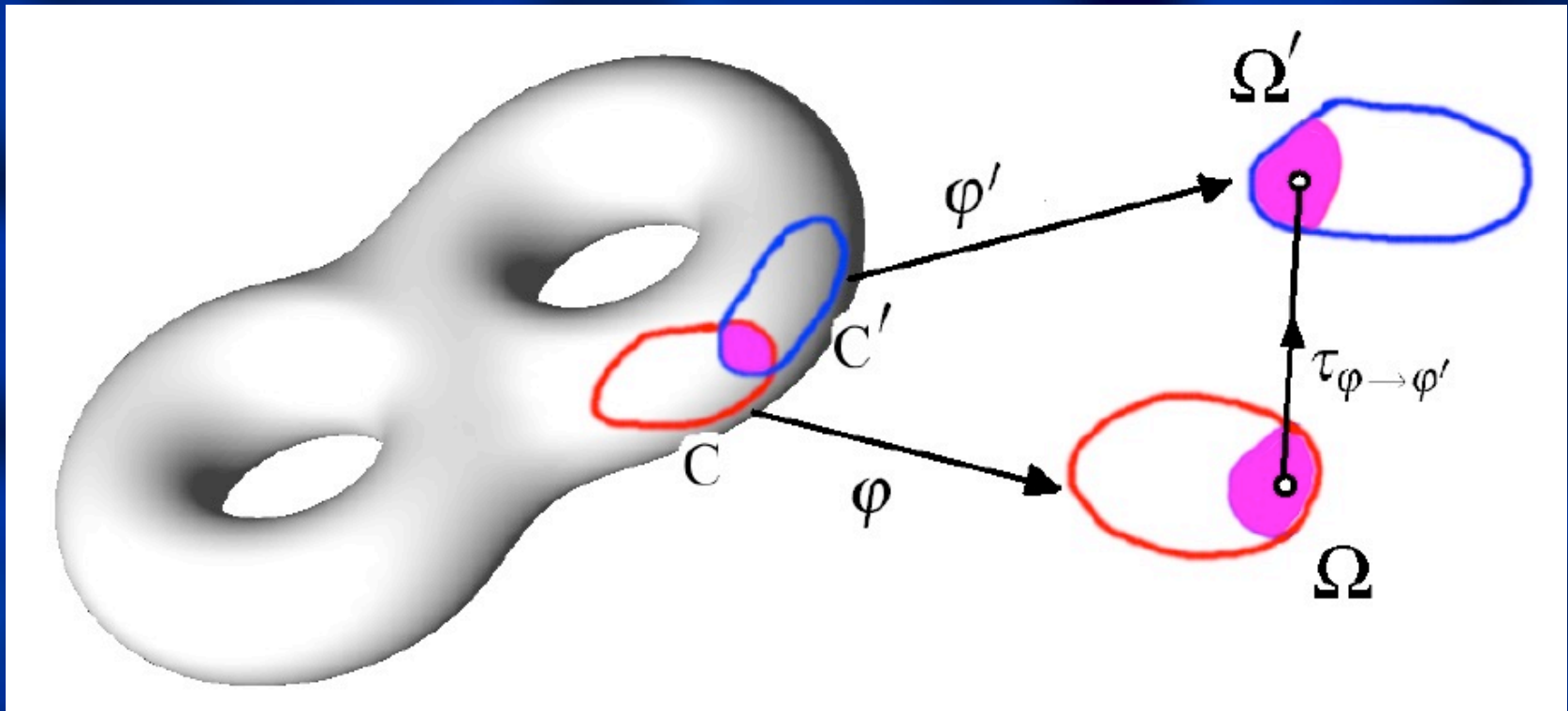
DEC version:
N-Symmetry
direction fields
[Ray et.al]

See also N-Rosy
[Zhang et.al]



3. Difficulties

Functions over manifolds

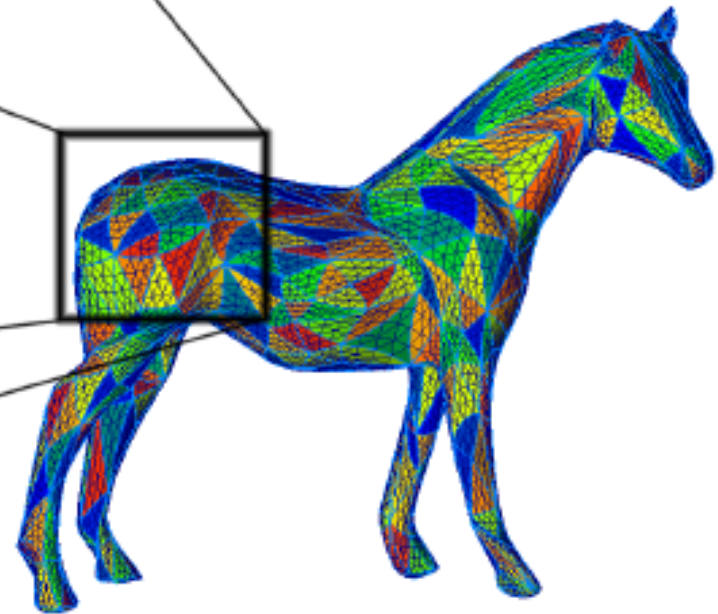
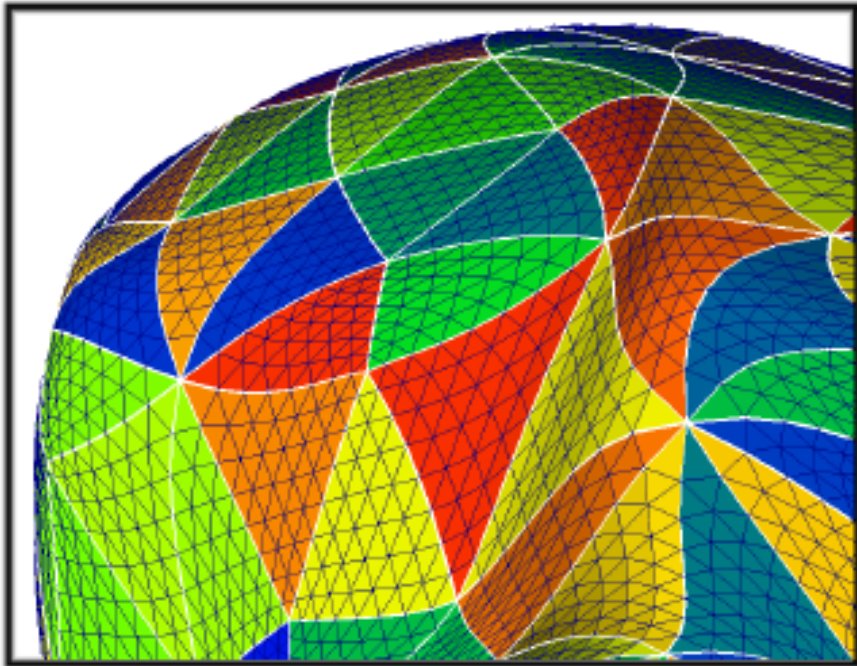


Differential Manifold

see, e.g. [Grimm et.al], [Gortler et.al], [Ray et.al]

3. Base Complex

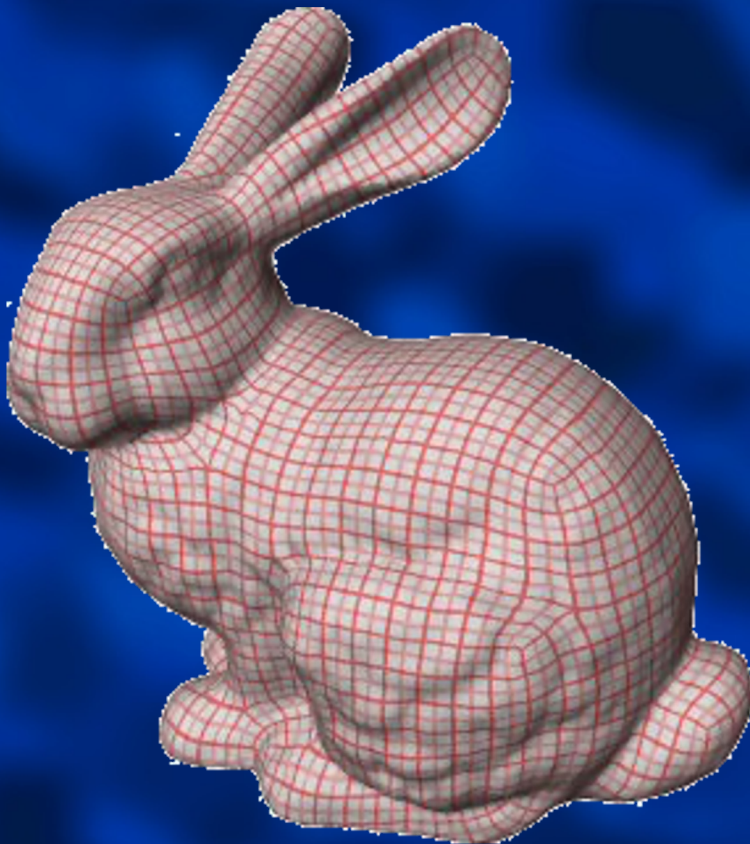
Triangulated Base Complex



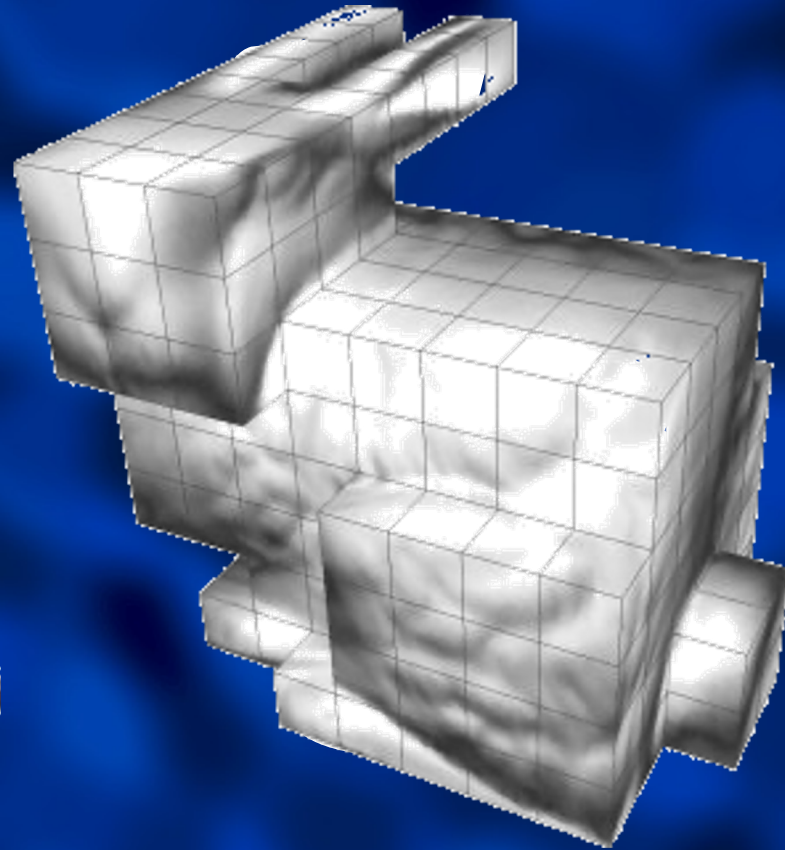
Multires Analysis, MAPS,
Inter-Chart Relaxation

3. Base Complex

Quadrangulated Base Complex



Polycubemaps
[Tarini et.al]

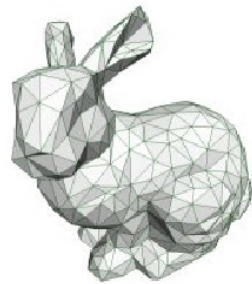


Semi-manual construction

2. Base Complex

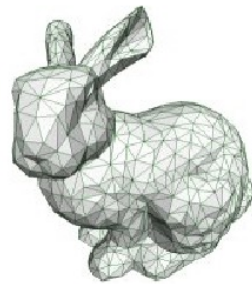
Why quads are desirable ?

+ tensor
product
splines



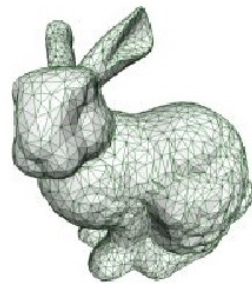
a geometry

+



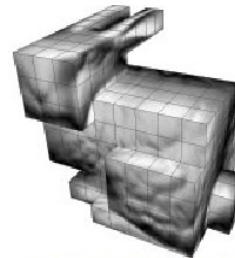
another geometry

+



vet another geometry

+

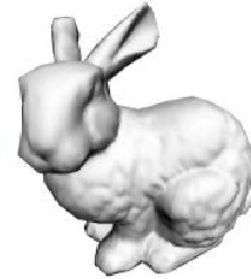


the same **PolyCubeMap**
(here storing pre-shaded normal values)

encoded in texture memory as:

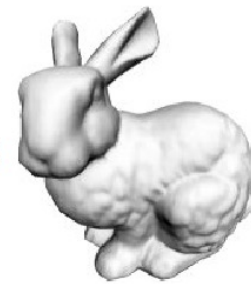


=



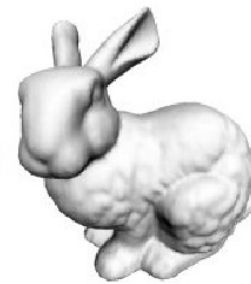
a good result

=



a good result

=

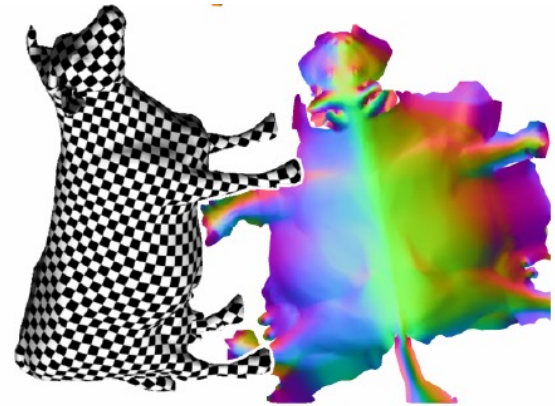


a good result

3. Base Complex

Frustration with charts

$$\begin{aligned} L_2(T) &= \sqrt{((1/\sigma_1)^2 + (1/\sigma_2)^2) / 2} \\ L_\infty(T) &= 1/\sigma_1 \end{aligned}$$



$$\mathbf{I_P} = \begin{bmatrix} \frac{\partial \mathbf{x}^2}{\partial u} & \frac{\partial \mathbf{x} \cdot \partial \mathbf{x}}{\partial u \partial v} \\ \frac{\partial \mathbf{x} \cdot \partial \mathbf{x}}{\partial u \partial v} & \frac{\partial \mathbf{x}^2}{\partial v} \end{bmatrix}$$

Lots of computations !
Handling charts is a hassle

Summary

- Porting Differential Geometry to the discrete setting [Pinkall&Polthier, Hormann&Greiner, Desbrun&Schroeder]
- Defining distortion
- Methods for disk topology
- Other methods: coming next...

Overview

This tutorial

1. Introduction

2. Differential Geometry on Meshes

Mesh Parameterization

3. Functions on Meshes

Discrete Exterior Calculus

----- 10h30 - 10h50: Coffee Break -----

4. Spectral Mesh Processing

5. Numerics